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ABSTRACT

This is a report of an experiment performed to evaluate the effects of the teaching of non-decimal numeration systems in grades four and six. Eighteen classes consisting of 430 students comprised the sample. The students were divided into four randomly assigned treatment groups for each level. Tests were given before a teaching session of five to six weeks, again directly following the teaching period, and once more as retention tests seven weeks after the conclusion of the teaching period. Equal arithmetic reasoning group mean scores were achieved by all groups on the posttest. The non-decimal groups of both grades had significantly higher group means on the retention test. Retention was greater among sixth graders than among fourth graders but this knowledge did not improve students' ability to answer questions on place value. Non-decimal group test scores on the posttest showed significant correlations with arithmetic reasoning and computation test scores. The recommendation is made that this topic should be taught in the upper elementary grades and that place value and the relation to the decimal system should be stressed. (Author/FL)

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FINAL REPORT

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A STUDY OF THE TEACHING OF NON-DECIMAL SYSTEMS OF NUMERATION IN THE ELEMENTARY SCHOOL

Phyllis F. Kavett, Ed. D.

Rutgers-The State University
New Brunswick, New Jersey

April 1969

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SUMMARY

Among the techniques used in the elementary school mathematics program to emphasize the number-numeral distinction is the teaching of non-decimal systems of numeration. An experiment to evaluate the effects of the teaching of non-decimal numeration in grades four and six was performed in Roselle and Elizabeth, New Jersey. Eighteen classes consisting of 430 students comprised the sample.

Teachers attended preparatory workshops of five weekly meetings. The student sample was divided into four randomly assigned treatment groups for each grade level. Three classes of each grade studied a unit on non-decimal systems; two classes studied a unit on intuitive geometry (non-computational); two classes studied a unit on the decimal system enriched by means of visual and manipulative aids; and two classes studied the regular program on the decimal system.

Tests were given before a teaching session of five to six weeks, again directly following the teaching period, and once more as retention tests seven weeks after the conclusion of the teaching period. The tests given included the California Test of Mental Maturity - Short Form - Level 2; Stanford Achievement Test - Arithmetic - Level I and II: Forms X and W - Arithmetic Computation and Concepts, and a Non-Decimal Test developed by the investigator for each grade.

Statistical Analyses

Analysis of variance and covariance were used for the arithmetic computation and reasoning tests. When a difference among group means was observed, the Scheffe Test was used to make comparisons.

A comparison of non-decimal test scores was made for each student's posttest and retention test scores using the Wilcoxon Matched-Pairs Signed-Rank test.

Intercorrelations among scores and other data were made using the Pearson Product-Moment Correlation.

Distributions of scores on the non-decimal tests were compared for the fourth and sixth grades with the Kolmogorov-Smirnov Two-Sample Test.

Findings

Equal arithmetic reasoning group mean scores were achieved by all groups on the posttest. The non-decimal groups of both grades had significantly higher group means on the retention test, possibly indicating a positive transfer effect caused by study of the topic.

Retention of knowledge of non-decimal systems of numeration was greater among sixth grade students than among fourth grade students.

This knowledge did not improve students' ability to answer questions on place value. Non-decimal groups test scores on the posttest showed significant correlations with arithmetic reasoning and computation test scores. Sixth grade students were generally more successful on the test of non-decimal systems than were the fourth grade students.

Intercorrelations of scores were slightly different for boys and girls, similar for different treatment and racial groups, and most dissimilar for groups separated according to teachers' judgments of degree of educational advantage.

Recommendations

Teaching of this topic in the upper elementary grades is recommended. Place-value and the relation to the decimal system should be stressed.

Further research might explore grade placement of the topic, merits of different methods of teaching, and long-range effects of learning the topic.

CHAPTER I

INTRODUCTION

Recent curriculum changes in mathematics education prompted Allendoerfer (1965) to stress the importance of research in the psychology of learning with application to mathematics. Earlier, Brownell (1961), had also called for this type of research because of innovations in content, grade placement, and instructional emphasis. Four types of change were classified by Brownell:

1) Traditional content was being taught earlier in the grades in order to spread learning over a longer period of time.

2) Emphasis was being placed on mathematical aspects of a topic in order to make that topic more meaningful.

3) Learning differences among children were being given greater consideration in order to individualize learning with variations in materials and teaching styles.

4) Content for the upper elementary grades was being modified to introduce topics such as approximation, mental arithmetic, statistics, number systems with bases other than ten, intuitive geometry, algebraic concepts, and casting out nines. In most instances, this content had never been previously associated with elementary school instruction (Brownell, pp. 66-69).

Many topics listed by Brownell have been incorporated into elementary school textbooks as part of the curriculum. However, experimental data to justify inclusion of some topics is minimal.

Learning the number-numeral distinction is a significant objective of elementary school mathematics. Instruction in computation is designed to show that a number may be named with a variety of numerals. One variation in assigning numerals is the use of non-decimal bases. The study of non-decimal systems of numeration has been widely accepted for classroom instruction (Smith, 1965). This acceptance can be traced to claims made by mathematics educators, inclusion of the topic in experimental programs, and inclusion of the topic in mathematics textbooks.

Yet, Fehr (1966) stated that there was a need for studies to support or refute the teaching of this mathematics topic in the elementary grades. He pointed out that "learning place systems in other bases, such as four, five, or seven, will help a child understand the decimal system better is a good hypothesis, but it has never been tested so far as the writer knows" (Fehr, 1966, p.84).

Statement of the Problem

Non-decimal systems of numeration are taught in the elementary school. This experiment was designed to investigate the effects of teaching non-decimal systems on the learning of decimal systems by fourth and sixth graders when computation and arithmetic achievement were used as evaluative criteria.

The experiment will seek answers to several questions:

Effects of Four Treatments on Criterion Measures

1) Will the learning of non-decimal systems of numeration have any effect on scores of tests of computation and arithmetic reasoning given immediately after the teaching period and on those tests given several weeks later?

2) Will the teaching of a non-numerical topic such as intuitive geometry affect scores on a standardized test of arithmetic achievement and reasoning?

3) Will the enriching of a regular arithmetic program with visual devices and nontextual materials affect scores on standardized tests of arithmetic achievement and reasoning?

4) Will the teaching of the usual arithmetic program of decimal numeration affect scores on standardized tests of arithmetic achievement and reasoning?

Effects of Study of Non-Decimal Systems

5) Will students who learned non-decimal systems of numeration retain this ability over a period of time?

6) Will the learning of non-decimal systems have any effect on scores of that portion of an arithmetic reasoning test containing questions on place-value and numeration?

7) Will significant positive correlations result on non-decimal test scores and scores on arithmetic computation and reasoning tests? Will the same students be successful on both types of tests?

8) Which grade level will be more successful in learning non-decimal systems of numeration, grade four or grade six?

Intercorrelations Among Groups

9) Will there be differences in test score intercorrelations among groups separated according to:

a) treatment group,

b) sex,

c) race,

d) degree of advantage?

Importance of the Study

Mathematics educators such as Deans (1963), Morton (1964), and Eicholz (1965) have pointed out that non-decimal systems can provide an interesting way to review, strengthen, and extend ideas of place-value. Other mathematics educators have rationalized inclusion of non-decimal systems into their textbooks after this was done in experimental programs such as SMSG. (SMSG, 1960). Subsequent textbook series have institutionalized the content and rationale for use in the elementary school.

Rahmlow (1965) summarized the trend by stating that "it is now common practice in many of the elementary schools to introduce the students to numeration in bases other than ten so they may appreciate and understand base ten more fully." (p. 339)

In contrast, some authors considered this topic to be useful enrichment at best and to be included at the teacher's discretion (Fehr and Philips, 1967).

An increase in understanding and appreciation of base ten numeration because of instruction in bases other than ten was predicted by Dutton (1961), Banks (1961), and earlier by Buckingham (1947).

Other educators emphasized student "interest and understanding" and indicated vocational and historical justifications. Grossnickle and Brueckner (1963) and Lovell (1964) referred to use in computers. Swain (1959) described computer use as plebeian but stressed use for statistical investigation, probability, and the analysis of strategy for games and puzzles.

Wren (1965) stated that "it is beneficial and of interest to review the struggles past civilizations have had with problems of base, place value, and the additive principle in the development of a numeration system." (pp. 21-22)

Keedy (1963) used the transfer principle as rationale when he declared:

> As a study of a foreign language aids one in understanding better his mother tongue, so a study of less familiar numeration can aid in understanding the familiar. (p. 14)

A more valid line of reasoning was used by Rappaport (1966) who pointed out that it was important for children to learn that a number has many names.

Emphasis on Hindu-Arabic numeration and place-value as elements in the study of non-decimal systems was expressed by Marks, Purdy, Kinney (1958), Mueller (1964), and Rudd (1963).

Wholey (1964), Osborne, DeVault, Boyd and Houston (1963), and Gibb (1959) discussed the need for instruction in non-decimal numeration in order to overcome the superficial understanding students have of decimal numeration.

Other mathematics educators wrote of the need for instruction in non-decimal numeration as preparation for more difficult topics in high school. The Report of the Cambridge Conference on School Mathematics (1963) stated the need for

> ...the explicit study of the decimal system of notation including comparison with other bases and mixed bases...in grades three through six in order to develop familiarity with the real number system and to start pre-mathematical experiences aiming towards more sophisticated work in high school. (pp. 36-37)

Creativity as a rationale was proposed by Osborn, Devault, et al. (1963) who maintained that "in the classroom today, the child may achieve a better understanding of our system of numeration if he is given an opportunity to create his own systems." (p. 21)

Crouch, Baldwin, and Wisner (1965), Jones (1958), and Brumfiel, Eicholz, and Shanks (1962) proposed similar rationales with emphasis on comparison of number system structure as an aid to understanding.

Inclusion of non-decimal topics in teacher education courses and related textbooks has become usual. Ruddell, Dutton, and Reckzeh (1960) announced that elementary school teachers in response to a questionnaire expressed the conviction that various operations in bases other than ten be taught to prospective teachers.

Mathematics educators also thought that instruction in non-decimal numeration was important in training prospective teachers. Corle (1964) stated that "a careful look at systems of notation other than the decimal system and to cast an appraising eye toward the future" (p. 71).

Ohmer, Aucoin, and Cortez (1964) stated that value was to be found in problems encountered by the teacher or parent when he studied a numeration system of base other than ten because these are "similar to the problems encountered by the child when he studies the Hindu-Arabic numeration system in arithmetic" (p. 131).

Newsom (1951) and Swenson (1964) declared that both teachers and students who have learned non-decimal numeration will enjoy a better understanding of base ten numeration.

Statements of rationale mention effects upon student interest and enthusiasm. Assertions have been made that interest and change in attitude resulted from study of non-decimal numeration. An implication is that transfer of interest occurred and decimal computation was improved. This assertion remains to be substantiated. Comparison of decimal and non-decimal numeration suggested to educators that automatic transfer occurred in a positive direction. Available research, however, indicated that positive or negative effects of non-decimal instruction on decimal computation remained to be demonstrated (Suydam, 1967).[1]

Definition of Terms

Decimal Numeration. Decimal numeration refers here to the naming of numbers using a positional system of numeration, the digits 0, 1,...9, and the base ten.

Non-Decimal Numeration. Non-decimal numeration is the naming of numbers using a base other than ten, the digits 0, 1,...n where n is the number of the base minus one, and a positional system.

Transfer. Transfer of learning is said to occur whenever the existence of a previously established habit has an influence on the acquisition, performance, or relearning of a second habit.[2]

Standardized Test of Computation. The standardized test of computation for this study consisted of Form X and Form W of the Stanford Achievement Test: Test 1 - Arithmetic Computation. Level I was used with the fourth grade. Level II was used with the sixth grade.

Standardized Test of Arithmetic Reasoning. The standardized test of arithmetic reasoning consisted of Form X and Form W of the Stanford Achievement Test: Test 2 - Arithmetic Concepts. Level I was used with the fourth grade. Level II was used with the sixth grade.

Intuitive Geometry. Intuitive geometry in this study consisted of units in topology and of units from Euclidian geometry, taught without use of formal proof.

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[1] Suydam (1967) conducted a survey of studies done in mathematics education for the period 1900-1965. Four studies were listed for the teaching of non-decimal numeration. Several unpublished doctoral studies have been conducted since 1965.

[2] J.A. McGeoch and A.L. Irion. The psychology of human learning. New York: Longmans, Green, 1952, p. 299.

Non-Textual Materials. Non-textual materials consisted of concrete objects and duplicated worksheets to assist or supplement the textbook.

Intelligence Quotient. Intelligence quotient was defined as the score achieved on the California Test of Mental Maturity - Short Form Level 2.

Disadvantaged Student. The disadvantaged student was defined as one judged by his teacher to have a lower than average educational expectancy, both in and out of school, attributable to social factors.

Teacher Workshop. Teacher workshops consisted of a series of five weekly two-hour meetings of teachers in each treatment group.

Treatment. A treatment was one of four prescribed curriculum units in mathematics with associated teaching methods developed for the experiment. These were:

1) Study of non-decimal systems of numeration (abbreviated - Non-Dec)

2) Study of intuitive geometry (abbreviated - Non-Comp)

3) Study of decimal system with emphasis on use of visual and manipulative devices (abbreviated - Dec-VM)

4) Study of regular decimal program with no change in method or sequence (abbreviated - Dec-Reg)

Test Administrator. The test administrator was a trained classroom teacher who administered all standardized tests to the students in this study.

Pretest. The pretest was the test period preceding the teaching period by approximately a week and consisted of intelligence and arithmetic achievement tests.

Posttest I. Posttest I was the test period directly following the teaching period and evaluated arithmetic achievement, non-decimal numeration, and intuitive geometry.

Posttest II. Posttest II consisted of tests given seven weeks after Posttest I and contained tests of arithmetic achievement, intelligence, and non-decimal numeration.

Visual and Manipulative Devices. Visual and manipulative devices consisted of concrete objects used by the learner to perceive mathematical relationships as well as aids used by the teacher to demonstrate or explain a mathematical concept.

Hypotheses

Data were collected and analyzed to test the following hypotheses concerning the grade four sample:

1) There are no significant differences for scores on STAN - Test 1, Form W (Posttest I) - Computation - among groups of fourth grade students receiving the four treatments.

3) There are no significant differences for scores on STAN - Test 2, Form W (Posttest I) - Arithmetic Reasoning - among groups of fourth grade students receiving the four treatments.

5) There are no significant differences for scores on STAN - Test 1, Form X (Posttest II) - Computation - among groups of fourth grade students receiving the four treatments.

7) There are no significant differences for scores on STAN - Test 2, Form X (Posttest II) - Arithmetic Reasoning - among groups of fourth grade students receiving the four treatments.

9) There are no significant differences for CTMM (Posttest II) scores when groups have been matched according to CTMM (Pretest) among fourth grade students.

11) There are no differences for difference scores between the Pretest and Posttest I STAN - Test 1 scores among groups of fourth grade students receiving the four treatments.

13) There are no differences for difference scores between the Posttest I STAN - Test 2 among groups of fourth grade students receiving the four treatments.

15) There are no significant differences for difference scores between the Posttest I and Posttest II STAN - Test 1 scores among the fourth grade students receiving the four treatments.

17) There are no significant differences for difference scores between the Posttest I and Posttest II STAN - Test 2 scores among the fourth grade students receiving the four treatments.

19) There are no significant differences for scores on the subportion of STAN - Test 2 directly testing the concept of place value and numeration among fourth grade students receiving the four treatments.

21) There are no significant correlations for fourth grade students separated according to sex and treatment among scores for intelligence, teacher judgment of arithmetic and reading ability, arithmetic computation; arithmetic reasoning, non-decimal numeration, and intuitive geometry.

23) There are no differences among fourth grade scores on the non-decimal tests (Posttest I) and (Posttest II).

The following hypotheses were tested in the analysis of data concerning the grade six sample:

2) There are no significant differences with respect to scores on STAN - Test 1, Form W (Posttest I) - Computation - among groups of sixth grade students receiving the four treatments.

4) There are no significant differences with respect to scores on STAN - Test 2, Form W (Posttest I) - Arithmetic Reasoning - among groups of sixth grade students receiving the four treatments.

6) There are no significant differences with respect to scores on STAN - Test 1, Form X (Posttest II) - Computation - among groups of sixth grade students receiving the four treatments.

8) There are no significant differences with respect to scores on STAN - Test 2, Form X (Posttest II) - Arithmetic Reasoning - among groups of sixth grade students receiving the four treatments.

10) There are no significant differences with respect to CTMM (Posttest II) scores when groups have been matched according to CTMM (Pretest) among sixth grade students.

12) There are no differences with respect to difference scores between the Pretest and Posttest I STAN - Test 1 scores among groups of sixth grade students receiving the four treatments.

14) There are no differences with respect to difference scores between the Pretest and Posttest I STAN - Test 2 among groups of sixth grade students receiving the four treatments.

16) There are no significant differences among the difference scores between the Posttest I and Posttest II STAN - Test 1 scores among the sixth grade students receiving the four treatments.

18) There are no significant differences among the difference scores between the Posttest I and Posttest II STAN - Test 2 scores among the sixth grade students receiving the four treatments.

20) There are no significant differences among scores on the subportion of STAN - Test 2 directly testing the concept of place value and numeration among sixth grade students receiving the four treatments.

22) There are no significant correlations for sixth grade students separated according to sex and treatment among scores for intelligence, teacher judgment of arithmetic and reading ability, arithmetic computation, arithmetic reasoning, non-decimal numeration, and intuitive geometry.

24) There are no differences among sixth grade scores on the Non-Decimal tests (Posttest I) and (Posttest II).

The fourth and sixth grade samples were combined in the testing of the following hypothesis:

25) There are no significant correlations for fourth and sixth grade students separated according to race and level of advantage among scores for intelligence, teacher judgment of arithmetic and reading ability, arithmetic computation, arithmetic reasoning, non-decimal numeration, and intuitive geometry.

The portion of the fourth and sixth grade samples in the non-decimal treatment were compared in the testing of the following hypothesis:

26) There are no differences among distribution of scores on the Non-Decimal tests (Posttest I and Posttest II) between the fourth and sixth grades.

Assumptions of this Study

1) Teachers had no knowledge of the experimental design or the questions being investigated.

2) All tests were administered under comparable classroom conditions.

3) Teachers were equally enthusiastic about participation in the project.

4) No teacher felt unduly pressured to complete any work unit. Completion date for any unit was determined by the teacher.

5) Effects due to teaching proficiency were distributed among treatment groups.

6) Children had no knowledge of participation in any special project.

7) Children had had no previous instruction in the topic randomly assigned to them.

8) No one group of children received significantly more outside help than any other group.

9) All children had equal access to special help or assistance from their teachers when any difficulty in learning was encountered.

10) The school districts of Elizabeth and Roselle, New Jersey, contiguous communities, were considered to have common content areas and curriculum sequences.

11) The same test administered to different groups over a scheduled interval of several school days was considered to be given during the same test period.

Limitations of this Study

1) The study was limited to a student population of 430 children; the number in the fourth grade almost equal to the number in the sixth grade.

2) Scores of children absent on testing days, or whose general absence was judged by teacher to be excessive, were eliminated.

3) Scores of children with foreign language or reading difficulties were omitted.

4) Effects of interest and attitude change towards mathematics were not considered in this study.

5) Evaluation in this study was limited to analysis of scores on standardized tests of arithmetic achievement, mental maturity, and specially-prepared tests of computation in non-decimal systems and intuitive geometry.

Review of the Literature

Rationales stated for inclusion of non-decimal systems in elementary school textbooks imply positive transfer, i.e., learning non-decimal systems will improve performance in decimal systems. Mention is hardly made of negative transfer, that form of proactive inhibition in which learning non-decimal systems may impede a student's ability to compute in the decimal system.

In non-decimal computation, a student would have to undergo the process of extinguishing competing responses in an example such as: $4_{five} + 2_{five} = 11_{five}$. The response "6" may have to be submerged.

Unfortunately, prior research dealing with non-decimal systems does not include mention of the transfer phenomenon. These studies do indicate, however, that further research is necessary before the claims of mathematics educators can be adequately substantiated.

Holmes (1949) taught two matched groups of seventeen seventh grade students. After four days of instruction he claimed evidence of attitudinal change as well as positive gain in understanding of decimal notation.

Hamilton (1961) taught number bases to prospective elementary school teachers by having them invent new symbols for numbers expressed in different bases. Test results between groups of teachers revealed no significant differences in understanding of the decimal system.

Hollis (1964) taught non-decimal numeration to one fourth grade class for seven school days and then reported pre- and posttest increases in median and mean test scores of arithmetic achievement. Hollis did acknowledge, however, the lack of randomness of his sample, the lack of statistical analysis, and the inadequacy of sample size.

Scott (1963) taught six kindergarten and eight grade one classes in a specially-selected sample. Assessment of student learning was done by Scott and the classroom teachers observing the experiment. Their collective judgment was that first graders had superior performance; a slight correlation might exist between performance and socio-economic level; and non-decimal topics might be introduced into kindergarten and grade one.

Lerch (1963) designated four fourth-grade classes in his experiment to study effects of non-decimal instruction on understanding of the decimal system. Two classes were assigned to serve as the control group and two classes served as the experimental group. An original short story entitled, "Numbers in the Land of Hand" was the basis for introduction and study of the topic in the experimental group. The quinary system was taught to the experimental group by Lerch during two extra periods a week for five weeks. Test results indicated a positive change in knowledge of the decimal system for the two experimental classes and a gain score in an investigator-devised test of attitude towards mathematics. Several factors have to be considered in the Lerch study:

1) The superposition of non-decimal numeration on a regular decimal system program raises the question whether gain or loss can be attributed to instruction in the regular decimal program or to non-decimal instruction

2) Use of the original story, "Numbers in the Land of Hand," created an interest factor which may be highly commendable from a motivational point of view, but which cannot be described as the usual or "expected" classroom approach for a topic in mathematics.

3) Extra sessions for the experimental group created a set of conditions which was not equal for the control group. The experimental group knew of its participation in an experiment.

4) Sample was not randomly selected and consisted of an experimental group N = 38 and control group N = 42.

McCormick (1965) carried out a comparative study of two methods of teaching decimal numeration. Using 177 fifth grade students divided into experimental and control groups, McCormick prepared non-decimal worksheets for eight 30-minute sessions. The experimental group used these worksheets during the regular decimal system program. No significant differences were reported with respect to improved understanding of the decimal system between sub-groups. McCormick stated, however, that a study of the means of the two groups indicated that the mean improvement of the decimal group was higher than that of the non-decimal group. He attributed this difference to the brief instruction in non-decimal systems. He cautioned that the brief instruction carried out in his study tended to confuse rather than clarify the thinking of a student.

Schlinsog (1965) carried out a study to determine the effects of supplementing sixth grade arithmetic with a study of other number bases. A series of tests was designed by Schlinsog to measure basic understandings of the decimal system, to check computational abilities, and to indicate change in preferences for arithmetic. Thirteen lessons in non-decimal systems were specially-prepared. These lessons were studied by four sixth-grade classes during their regular mathematics program. Other classes studied the decimal system with no change in program. Aside from attitudinal changes, Schlinsog reported no significant differences between those studying non-decimal systems and those in the regular program. He pointed out that thirteen lessons could be considered highly inadequate and results might have been different if more time had been available.

Jackson (1965) carried out a study of effects of instruction in non-decimal systems on "selected objectives of mathematics education." He prepared worksheets and student units for groups studying decimal and non-decimal systems. Teachers in both groups were given a teacher's guide outlining procedures to follow and content to be taught. Classroom instruction with these units lasted four weeks.

Similar content appeared in units prepared for both groups. These topics were: Historical Development of Numeration Systems, Development of Decimal Systems of Numeration, and Meaning of Place Value. Topics discrete for the different groups were:

Non-decimal Group: Base Five, Base Twelve, Base Two, and a short unit on computers.

Decimal Group: Meaning of Addition, Subtraction, Multiplication, and the reading and writing of numerals representing large numbers. (Pp. 48-50)

Jackson acknowledged that a clear distinction did not exist between the units prepared for each group at each grade level.

He reported that fifth grade pupils studying non-decimal systems tested significantly better than the decimal group on the nature and operation of the decimal system. Seventh grade pupils showed no significant differences between groups. On tests of the nature of numeration systems the fifth grade students studying the decimal system did better than the non-decimal group and seventh grade students studying non-decimal systems did better than the decimal group.

The literature describing studies of effects of non-decimal instruction on decimal system operations has been non-conclusive. Acknowledgment of the transfer phenomenom does not appear. Yet, many mathematics educators stress the need for further investigation of transfer as it applies to mathematics learning (Rosskopf, 1953; Shulman, 1967; Becker and McLeod, 1967). Where research in non-decimal systems did exist, it was marked by several characteristics:

1) Small, non-random samples were used.

2) Instructional periods were of relatively short duration.

3) The experimenter often served as classroom teacher and evaluator.

4) Other topics in mathematics were studied concurrently, and

5) Statistical controls were often lacking.

This study attempted to overcome the shortcomings of earlier research. Several conditions were therefore included in the research design.

These were:

1) A sample of 18 teachers was randomly selected from 45 teachers who indicated willingness to participate in this study.

2) Instruction of teachers preceded any classroom try-out.

3) The sample consisted of 430 students divided between grades four and six.

4) No mathematics topics, except those specially-prepared for this experiment were taught to students.

5) Classroom instruction of any topic lasted at least five weeks.

6) Statistical design included pretests, posttests, and tests of recall.

7) Control groups were established. Three different treatments were randomly assigned to these groups:

(a) A non-computational unit in Intuitive Geometry.

(b) A decimal system unit enriched with specially-prepared visual and manipulative aids, and

(c) Maintenance of the regular decimal system program.

These treatments were devised to preclude the possibility of differences due to a Hawthorne effect because of the newness of the topic, non-decimal systems of numeration, or to the stimulation of interest resulting from use of special materials and methods.

8) Analysis included comparisons of arithmetic achievement, socioeconomic level, estimates of reading and arithmetic achievement, and intelligence quotient.

CHAPTER II

DESCRIPTION OF THE EXPERIMENT

Sample

Communities

The school districts of Roselle and Elizabeth, New Jersey were selected for this study. There were many reasons for this decision:

1) Teachers and administrators of both school districts had expressed an interest in classroom research.

2) Non-decimal systems of numeration had not been included as part of the regular elementary school mathematics curriculum in these two school districts. In rare instances, teachers had included the topic in their classroom program.

3) The communities are adjacent to each other and are heterogeneous with respect to racial, ethnic, and socio-economic backgrounds.

4) Many teachers voluntarily agreed to participate in an experiment of mathematics teaching.

5) The eighteen teachers who were randomly selected to participate in the study agreed to concentrate on their assignment whichever one it might be. They had no knowledge of the research design.

6) The participating teachers had not taught non-decimal systems of numeration that same school year.

Grade Level

Non-decimal systems of numeration usually appear in elementary school mathematics textbooks on the fourth grade level. Several series begin non-decimal systems in grade six. Grades four and six were therefore selected for this study.

Maturational factors were also considered. Grades four and six seemed sufficiently spaced to allow differences in growth and achievement levels to influence experimental data.

Teachers

Teachers of the fourth and sixth grades were invited to participate in a study dealing with mathematics teaching. Forty-five teachers applied. A random selection was made and nine teachers from grade four, and nine teachers from grade six participated. Those teachers designated as "standby" teachers were not called upon. All eighteen teachers beginning the study completed their part satisfactorily.

Students

The student sample for this study consisted of 430 students distributed among eighteen participating classes; nine from grade four and nine from grade six.

Table 1 indicates the number of boys and girls for grades four and six.

TABLE 1

Sex and Grade of Student Sample

Grade	Boys	Girls
Four	101	107
Six	128	94

Distribution according to sex was about equal in grade four. In grade six, however, the number of boys was greater than the number of girls.

Teacher participants were asked to identify workshop day preferences and were then randomly assigned to workshop groups. Treatments were also randomly assigned to these groups. Two classes from each grade level participated in each of the four treatments. The non-decimal system treatment consisted of three classes from each grade level, however. Table 2 shows the distribution of students among treatment groups.

TABLE 2

Students by Grade According to Treatment Group

Treatment	Grade Four	Grade Six
Non-Dec	66	70
Non-Comp	52	53
Dec-VM	42	54
Dec-Reg	48	45

Teachers were asked to rate students according to a socio-economic scale based on personal judgment and student record cards. Three broad categories were selected: Advantaged, Normal, and Disadvantaged. Some criteria for discrimination were the following: stability of the home situation, parent's source of income, and out-of-school opportunities available to children.

Table 3 shows the socio-economic distribution of students in this experiment.[1]

TABLE 3

Teacher Judgments of Student Population
Based on Socio-Economic Criteria

Status	Number of Students	Percent
Advantaged	61	14.2
Normal	327	76.0
Disadvantaged	42	9.8

About fifty percent of the Black children who participated in the study were in predominantly Black schools. The other fifty percent were distributed throughout the schools of Roselle and Elizabeth. The children classified as "disadvantaged" were largely from the Black community. Table 4 shows the percentage distribution of students according to information supplied by teachers.

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[1] Consideration of socio-economic background is a result of recent awareness of social interaction processes in and out of the classroom. Reference is made to Romberg and DeVault (1967) who stated that research must be undertaken in actual classrooms and that influence of variables be acknowledged; and to Hungerman (1967), Fisher (1966), Baker (1966), and Rokeach (1960) who stated the importance of socio-economic background in classroom research dealing with mathematics achievement, cognitive behavior, and personality patterns.

TABLE 4

Number of Children According to Racial Designation

Racial Designation	Number	Percent
White	357	83.0
Black	71	16.5
Other	2	.1

These descriptions of the student sample included only those students whose data were available for the study. Test scores of children with foreign language difficulties or sensory disorders were not included in the analysis. Data for those children judged by the teacher to have excessive absence were also omitted from analysis. Children who missed one or more tests during the three test periods constituted the largest group of omissions. In all, data for about sixty students were eliminated from analysis.

Teacher estimates were recorded of student grade levels in reading and arithmetic. These estimates were converted into categories of low, normal, and high, corresponding to below grade level, at grade level, and above grade level.[1]

Conduct of the Experiment

The experiment took place during the Spring semester of the school year 1967-1968. Teacher participants were notified of their acceptance into the research program late in January 1968.

Teacher workshops began during the first week of February and lasted until the middle of March.

Pretests were given during February.

Experimental teaching period, begun during the last week of February, continued for six weeks through the month of March.

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[1] The researcher is aware of the limited faith which may be placed in these estimates. Worms (1966) showed that a 40% accuracy may exist in identification of slow learners and gifted children.

The first posttest period (Posttest I) took place in April. These tests preceded a Spring school recess of approximately ten days.

The second posttest period (Posttest II) took place seven weeks after the first posttest period. These posttests were considered tests of retention.

All testing was completed by June 15.

Teacher Workshops

Participating teachers attended five afternoon workshops, each of which lasted approximately two hours. These instructional sessions were directed by the researcher on different days of the week.

These workshops allowed for discussion of content for each experimental treatment in order to minimize effects or deficiencies in each teacher's earlier training and experience.

The workshops enabled these teachers to prepare to teach concepts not previously taught by them and possibly improve their skill at teaching familiar topics. Teachers' attitudes toward mathematics and the research project were improved as they learned more about their assigned topics (Williams, 1966).

Moreover, Schumann (1964) pointed out that among prerequisites for content change by teachers is a sincere willingness to work with a qualified consultant and evaluator. The use of a handbook alone without a consultant often may not result in teacher improvement.

In some cases, classroom materials were designed by the teacher participant after consultation with others in the group, and particularly with his grade-partner. The researcher acted as workshop leader and answered questions about content and teaching of elementary school mathematics.

Manipulative devices and student worksheets were demonstrated by the teachers and the researcher. The teachers, however, made their own selections of materials for classroom use. Teachers were encouraged not to change their teaching style nor to indicate participation in an experiment to their students.[1]

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[1] In an experiment involving teaching of science to fifth and sixth graders, Brudzynski found that concept achievement showed slight variation due to teaching styles. Whereas the lecture-demonstration techniques surpassed the inductive methods on first trial tests, for delayed retention, the style made little difference (Brudzynski, 1966).

Workshop meetings were held in locations central for each group of teachers. After-school travel time was held to a minimum.

Workshop agendas and management procedures for each treatment group were identical. Mathematics content alone marked the differences in each workshop session.

Workshop sessions and classroom implementation overlapped, thereby giving the teachers an opportunity to exchange classroom feedback and to pace their teaching.

No limits were given for classroom teaching of any portion of the prescribed topic. Each teacher was permitted to decide when coverage of a topic had been adequate. A check of teachers' daily records indicated variations were slight and an extreme case would be one week's difference. Posttests were scheduled according to each class' completion of its topic.

Research Design

The design for this experiment in transfer may be symbolized as follows:

	Before Experiment	Experiment Teaching Period	Before Retention Tests
1)	A	B	A
2)	A	$\sim A$	A
3)	A	A_1	A
4)	A	A	A

In this diagram, A represents decimal computation in the regular program. B represents the study of non-decimal systems. $\sim A$ represents the non-computational program, and A_1 represents the decimal system taught with visual and manipulative aids. The diagram indicates that retention tests followed the return to a regular decimal program by all groups.

Description of the Four Treatments

1) Non-Decimal Systems: Teachers of the six classes using this treatment taught the following subjects:

(a) Meaning of non-decimal numeration

(b) Notation of non-decimal numerals

(c) Addition and subtraction of non-decimal numerals
(d) Conversions of numerals expressed in base ten to numerals expressed in base five and the reverse
(e) Multiplication of non-decimal numerals (grade six only)
(f) Optional consideration of non-quinary bases.

Visual and manipulative instructional devices were used: tables of operation, simple odometer, abacus, pictorial symbols, small concrete objects, dittoed and mimeographed worksheets, and overhead projector transparencies.

Teachers were permitted to select those materials deemed appropriate for the learning style of their classes. Materials and suggested methods were modeled after descriptions in contemporary arithmetic tests for grades four and six.

Suggestions for materials and worksheets were found in articles by Greenholz (1964), Hilaire (1964), Hughes (1964), Karlin (1965), Nechin and Brower (1959), Ochsenhirt and Wittermeyer (1963), Rabinowitz (1966), Schupback (1967), and Weyer (1967). (Appendix A)

2) Non-Computation, Intuitive Geometry: Teachers of the four classes in this treatment used specially prepared materials. Units were prepared by senior college students as part of a course requirement. The best of these materials was selected by the teachers.[1]

This unit included the following subtopics for the fourth grade:

(a) Points, curves, regions, and planes
(b) Simple and complex curves
(c) Recognition and properties of some geometric forms, informal definitions
(d) Area puzzles-tangrams
(e) Use of geometry tools
(f) Construction of simple figures
(g) String constructions and curve stitching

The unit designed for grade six included the following subtopics:

(a) Closed curves and plane regions
(b) Construction of regular polyhedra
(c) Line drawings of solid figures - cylinder cone, cube, triangular prism, and square pyramid

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[1] The four classes in this experimental treatment became enthusiastic about the study of geometry and responded favorably to the prepared units. The four teachers, who at first were hesitant at postponing "number work" for a lengthy period of time, later were content with the favorable results.

(d) Cube and tetrahedron puzzles
(e) Properties of the cube, cylinder, cone, sphere, and pyramid
(f) Symmetry

An "End of Unit" test was developed for each grade level so that teachers were able to evaluate students' performance.

3) Decimal System: Visual-Manipulative Emphases: Four classes continued their regular sequence in mathematics. This program was enriched, however, with visual and manipulative instructional aids.

Fourth grade content during the classroom teaching session consisted of multiplication and division of whole numbers. Visuals and objects prepared for these classes at this time were:

(a) Overhead projector transparencies
(b) Play Tiles (resembles GeoBoard with plastic squares and rectangles for insertion into regularly placed holes)[1]
(c) Felt board cutouts
(d) Colored chalk for chalkboard
(e) Abacus for base ten numeration
(f) Simple odometer
(g) Plastic discs, tongue depressors, buttons.

Sixth graders were beginning the study of decimal fractions. After work with conversions from decimal fractions to common fractions, the four basic operations were considered in the usual order: addition, subtraction, multiplication, and division.

Teaching aids provided for these classes included:

(a) Graph paper
(b) Colored chalk and chalkboard
(c) Overhead projector transparencies
(d) Felt board cutouts
(e) Abacus
(f) Dittoed worksheets
(g) Bulletin board materials

4) Regular Program: The mathematics content of the fourth and sixth grade programs in this treatment was the same as that of the previous treatment.

The four teachers in this treatment, who were also advised to maintain their regular program, attended workshop sessions. Participation in these meetings enabled each gradepair of teachers to teach curriculum topics simultaneously. Discussion of the teaching process, analysis of feedback from the classroom, and concern for individual student problems, gave these teachers a sense of involvement in the experiment.

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[1] Play Tiles, Halsam Co., Chicago, Illinois (mod.)

Table 5 shows the number of classes in each grade level assigned to each treatment group.

TABLE 5

Number of Classes by Grade in Each Treatment

	Non-Dec	Non-Comp	Dec-VM	Dec-Reg
Grade Four	3	2	2	2
Grade Six	3	2	2	2

Testing Program

Grade Four Tests

All fourth grade classes were given the following test:

Pre-Test of Vision, Hearing, and Motor Coordination:
California Test Bureau (Pretest VHMC)

Children with sensory problems were identified by this means.

Grade Four and Grade Six Tests

All fourth and sixth grade classes were tested with appropriate grade level forms of the following tests:

California Test of Mental Maturity, 1963 Revision,
Level 2 Short Form (CTMM)

Stanford Arithmetic Achievement Test, 1964
Revision, Form W, Form X (Stan W, Stan X)

Test 1: Arithmetic Computation;
Test 2: Arithmetic Concepts

Special Tests

Non-Decimal Numeration Test (see Appendices B, C, and D). A test was developed for each grade using the Non-Decimal treatment.

End of Unit: Geometry Test. A test was developed for each grade using the Non-Computation treatment.

Test Administration

Tests were administered according to the time sequence described in Duration of the Study. Tables 6 and 7 describe the order in which specific tests were given to each treatment group.

TABLE 6

Grade Four Test Sequence According to Treatment

	Pretest	Posttest I	Posttest II
Non-Dec	Practice exercise for IBM answer sheet Pretest VHMC CTMM Stan X	Non-Decimal Numeration Test Stan W	Non-Decimal Numeration Test CTMM Stan X
Non-Comp	Practice exercise for IBM answer1 sheet Pretest VHMC CTMM Stan X	End of Unit; Geometry Test Stan W	CTMM Stan X
Dec-VM	Practice exercise for IBM answer sheet CTMM Stan X	Stan W	CTMM Stan X
Dec-Reg	Practice exercise for IBM answer sheet Pretest VHMC CTMM Stan X	Stan W	CTMM Stan X

TABLE 7

Grade Six Test Sequence According to Treatment

	Pretest	Posttest I	Posttest II
Non-Dec	CTMM Stan X	Non-Decimal Numeration Test Stan W	Non-Decimal Numeration Test CTMM Stan X
Non-Comp	CTMM Stan X	End of Unit: Geometry Test Stan W	CTMM Stan X
Dec-VM	CTMM Stan X	Stan W	CTMM Stan X
Dec-Reg	CTMM Stan X	Stan W	CTMM Stan X

Psychometric Characteristics of Tests

The psychometric characteristics of tests used in the study are shown in Tables 8 and 9 for grades four and six, respectively.

TABLE 8

Psychometric Characteristics of Tests Used in Study - Grade Four

Test	No. of Items	Test Time in Minutes	Form	N	Reliability		
					KR_{20}	KR_{21}	S-B*
CTMM, Level 2	120	43	Short	Manual		.95	
Stanford Arithmetic Level I	39	35	X,W	Manual	.86		
Test 1, Arith Comp.							
Pretest	39	35	X	208		.69	
Posttest I	39	35	W	208		.78	
Posttest II	39	35	X	208		.76	
Level I							
Test 2, Arith Concepts							
Pretest	32	20	X,W	Manual	.87		
Posttest I	32	20	X	208		.70	
Posttest II	32	20	X	208		.79	
Place-Value Subtest**							
Pretest	7		X	208	.41	.28	.78
Posttest I	7		W	208	.63	.59	.90
Posttest II	7		X	208	.40	.20	.77
Non-Decimal Test							
Posttest I	41	Untimed		20	.91		
				23	.94		
Posttest II	41	Untimed		20	.84		
				23	.90		
Vision Pretest	40	4					
Hearing Pretest	15	Untimed					
Motor Coordination Pretest	20	Untimed					

*Spearman-Brown Prophesy Formula is an estimate of a full-length test consisting of similar questions. This reliability is estimated for a test five times as long.

**This Subtest consisted of questions #1,13,11,15,17,21, and 24 for Form X and #17,1,6,23,16,20,27 for Form W of the Stanford Arithmetic Test 2, Level I.

TABLE 9

Psychometric Characteristics of Tests Used in Study - Grade Six.

Test	No. of Items	Test Time in Minutes	Form	N	Reliability		
					KR_{20}	KR_{21}	S-B*
CTMM, Level 2	120	43	Short	Manual		.95	
Stanford Arithmetic Level II	39	35	X,W	Manual	.87		
Test 1, Arith Comp.							
Pretest	39	35	X	222		.74	
Posttest I	39	35	W	222		.80	
Posttest II	39	35	X	222		.82	
Stanford Arithmetic Level II	32	20	X,W	Manual	.87		
Test 2, Arith Concepts							
Pretest	32	20	X	222		.75	
Posttest I	32	20	W	222		.81	
Posttest II	32	20	X	222		.83	
Place-Value Subtest**							
Pretest	8		X	222	.48	.35	.82
Posttest I	8		W	222	.68	.60	.91
Posttest II	8		X	222	.61	.51	.89
Non-Decimal Test							
Posttest I	41	Untimed		24 23	.94 .88		
Posttest II	41	Untimed		24 23	.87 .88		

*Spearman-Brown Prophesy Formula is an estimate of a full-length test consisting of similar questions. This reliability is estimated for a test five times as long.

**This Subtest consisted of questions #1,2,3,4,7,14,22,24 of Form X and #3,5,4,14,12,22,16, and 26 of Form W of the Stanford Arithmetic Test 2, Level II.

In Tables 8 and 9 are shown the reliabilities for the standardized tests as reported in the publisher's test manuals and also the reliability for the sample used in this study.

Statistical Analyses

Data derived from the tests described in the Test Program were collected and analyzed. Test scores and other identification data of each student in the experiment formed thirty-seven "variables" by means of which statistical analyses were made (Appendix E). Each student's data were punched on IBM cards.

Six basic types of analyses were performed to test the hypotheses:

1) Comparisons of group means on standardized tests of intelligence and arithmetic computation and reasoning.

2) Comparisons of score differences on arithmetic tests given during pretest, posttest I, and posttest II (retention) test periods.

3) Comparison of group mean scores on place value sub-tests of arithmetic reasoning tests.

4) Comparison of the distribution of scores of the fourth and sixth graders on the non-decimal systems test.

5) Intercorrelations of scores among students grouped by various identifying characteristics such as treatment, sex, grade, and race.

6) Comparison of posttest I and posttest II scores of each student on the non-decimal test.

Statistical Procedures

Analysis of Variance and Covariance. The analysis of variance and covariance was the statistical procedure used for categories (1), (2), and (3) of the above list. The following underlying assumptions were checked statistically wherever possible:

1) Homogeneity of within-group variance.

2) Homogeneity of within-group adjusted variances.

3) Linearity of the overall regression line, including:

   (a) equality of the within-group regression coefficients,
   (b) linearity of between-class regression,
   (c) equality of between-class regression and within-class regression (Dixon and Massey, 1957; and Winer, 1962).

4) Existence of non-zero regression coefficients.

Where there proved to be significant differences among the means for each treatment group, the Scheffe Test was employed to compare treatment groups and certain combinations of treatments.

Intercorrelation Analysis. The Pearson Product-Moment correlation coefficient was obtained for selected variables listed in Appendix E.

The test of significance of a correlation coefficient was based on the assumption that if two variables bear no relation to each other, their correlation coefficient *r* would be zero. Therefore, the *r* must be sufficiently different from *0* to be considered significant. Further, as the number in the sample increased, the *r* might be of a lower value to be considered significant. The computation of significant *r*'s was based on the following relation:

$$r = \frac{t}{\sqrt{t+N-2}}$$

Tabled values are found in many statistics textbooks. Additional values needed for analysis are shown in Appendix F.

Comparison of Fourth and Sixth Grade Scores on Test of Non-Decimal Systems. The Kolmogorov-Smirnov Two Sample Test (Appendix G) was used to compare distributions of scores of the two fourth grade classes with the scores of the two sixth grade classes on each of the Non-Decimal Tests. (Refer also to Siegel, 1956, p. 131).

Comparison of Scores for Each Student on Test on Non-Decimal Systems. The Wilcoxon Matched-Pair Signed-Rank Test was used to compare each grade group's performance on the non-decimal test given as posttest with that given as retention test. (Wilcoxon, Katti, Wilcox, 1963; Siegel, 1956, p. 75).

CHAPTER III

RESULTS FOR GRADE FOUR

Pretest Data

Evaluation of the teaching of non-decimal systems of numeration was carried out with four experimental teaching treatments, each preceded by tests yielding intelligence test quotients and arithmetic achievement scores. These pretest scores were used as covariants in order to equate treatment groups statistically, that is, to eliminate sources of differences in treatment means resulting from earlier experiences.

The means and standard deviations for the treatment Non-Dec on various test instruments are shown in Table 10. The scores are shown with and without inclusion of the test constructor class in order to demonstrate the representative nature of these classes. As may be observed, differences between the sets of data representing the groups with and without the test constructor sections are very small.

Means and standard deviations for the Non-Comp, Dec-UN, and Dec-Reg treatment groups are presented in Table 11.

TABLE 10

MEANS AND STANDARD DEVIATIONS ON TEST INSTRUMENTS FOR GRADE FOUR - TREATMENT NON-DEC

	Without Test Constructor Class N=43		With Test Constructor Class N=63	
	Mean	S.D.	Mean	S.D.
CTMM (Pretest)	108.67	12.62	109.14	13.20
CTMM (Posttest)	111.37	15.61	111.89	15.35
Stanford Test 1-X (Pretest)	13.42	5.31	13.35	4.84
Stanford Test 1-W (Posttest I)	16.72	6.57	16.47	5.80
Stanford Test 1-X (Posttest II)	18.30	6.74	17.42	6.42
Stanford Test 2-X (Pretest)	11.56	5.65	11.65	5.57
Stanford Test 2-W (Posttest I)	14.21	6.26	14.18	6.13
Stanford Test 2-X (Posttest II)	15.53	6.32	14.71	6.41
Non-Decimal Test (Posttest I)	24.72	10.95	23.53	14.71
Non-Decimal Test (Posttest II)	22.44	11.41	23.63	10.83
Place-Value Subtest (Pretest)	3.28	1.39	3.44	1.55
Place-Value Subtest (Posttest I	4.25	1.96	4.32	1.86
Place-Value Subtest (Posttest II)	3.67	1.39	3.71	1.48

TABLE 11

MEANS AND STANDARD DEVIATIONS ON TEST INSTRUMENTS GRADE FOUR - TREATMENTS NON-COMP, DEC-VM, DEC-REG

	Treatment Non-Comp N=52		Treatment Dec-VM N=42		Treatment Dec-Reg N=48	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
CTMM (Pretest)	104.86	13.70	93.21	12.84	107.46	13.46
CTMM (Posttest)	108.69	14.34	92.62	13.94	112.96	13.89
Stanford Test 1-X (Pretest)	13.23	4.53	11.17	5.27	15.79	5.41
Stanford Test 1-W (Posttest I)	14.29	5.23	13.76	6.47	20.67	5.71
Stanford Test 1-X (Posttest II)	16.19	5.28	13.31	4.93	18.84	6.01
Stanford Test 2-X (Pretest)	10.17	4.44	8.79	3.09	12.50	4.12
Stanford Test 2-W (Posttest I)	12.54	4.82	10.50	4.39	15.81	5.50
Stanford Test 2-X (Posttest II)	13.08	4.89	9.90	4.33	16.23	5.33
Place-Value Subtest (Pretest)	3.21	1.51	2.55	1.52	3.64	1.26
Place-Value Subtest (Posttest I)	4.00	1.90	3.17	1.70	4.35	1.79
Place-Value Subtest (Posttest II)	3.57	1.47	3.24	1.41	4.14	1.35
Geometry Unit Test	15.46	4.57				

An examination of Tables 10 and 11 reveals that students in Treatment Dec-VM had scores on pretests which were consistently lower than those of the other groups.

The Hartley Max-F test of homogeneity was applied to the variances given in Tables 10 and 11 in order to determine the appropriateness of analysis of variance to test for equality of treatment means. Table 12 presents the data of the test for homogeneity of variance.

Certain percentage points of the variance ratio for the Hartley Test may be seen in Appendix H. Table 12 reveals that, but for the STAN-Test 2, Form X. (Pretest) - Grade Four, analyses of variance are highly appropriate.

Slight departure from homogeneity of variance also occurred in the scores of the sixth grade CTMM (Pretest) and the STAN-Test 1, Form W, (Posttest I).

However, analysis of variance was performed for all the scores without exception for the following reasons:

1. the F-test is robust with respect to small departures from homogeneity, and

2, there is a slight bias toward rejection of the hypothesis of homogeneity because with unequal groups the larger N is used. (Winer, 1962, p. 94)

The primary purpose in collecting scores on these pretests was their later use as covariants in posttest and retention test analyses.

TABLE 12

HARTLEY MAXIMUM-F TEST OF HOMOGENEITY OF VARIANCE OF TEST SCORES OF FOURTH GRADE TREATMENTS

	Largest Variance	Smallest Variance	Numbers in Both Groups	$\frac{s^2\text{max}}{s^2\text{min}}$	$F_{.05}$**	$F_{.01}$***
CTMM (Pretest)	187.7	159.3	52.43	1.18	2.06	2.5
CTMM (Posttest)	243.7	192.9	43.48	1.26	2.12	2.6
Stanford Test 1-Form X (Pretest)	29.3	20.6	48.52	1.42	2.06	2.5
Stanford Test 1-Form W (Posttest I)	43.2	27.4	43.52	1.58	2.06	2.5
Stanford Test 1-Form X (Posttest II)	45.4	24.3	43.42	1.87	2.21	2.7
Stanford Test 2-Form X (Pretest)	32.0	9.6	43.42	3.33***	2.21	2.7
Stanford Test 2-Form W (Posttest I)	39.2	19.3	43.42	2.03	2.21	2.7
Stanford Test 2-Form X (Posttest II)	39.9	18.7	43.42	2.13	2.21	2.7
Place-Value Subtest (Posttest I)	3.84	2.89	43.42	1.33	2.21	2.7
Place-Value Subtest (Posttest II)	2.16	1.82	52.48	1.19	2.06	2.5

Analysis of Variance and Scheffe Test for Comparisons among Means

The California Test of Mental Maturity, given during the pretest period, was analyzed by means of the derived data shown in Table 13.

TABLE 13

ANALYSIS OF VARIANCE CTMM (Pretest)
(N=185)

Source	SS	df	Mean Square	F Ratio
Among	6429	3	2143	12.298^{***}
Within	31542	181	174	
Total	37971	184		

Sums of squares of the total sample, the total variation, is shown as well as the two parts into which each was divided. The first part, labelled "Among," is the sum of squares variation due to the deviation of the means of each treatment from the total mean.

The second part, labelled "Within," is due to the deviation of each score in a treatment from the mean of that treatment.

Hereafter in this study, the level of significance .01 will be indicated in all tables by three asterisks; the level .05 by two asterisks; the level .10 by one asterisk; and the level .25 by one number sign, #. The latter two levels of significance were used only in the Scheffe Test (Appendix I).

The F-ratio in Table 13, significant to the .01 level, indicated that the treatment means were unequal. Therefore, the Scheffe Test was employed to verify comparisons between the six pairs of means, two additional comparisons were made by this method:

1. Treatment Non-Comp was compared with the weighted mean of scores in Treatments Non-Dec, Dec-VM, and Dec-Reg; the one group not doing numerical computation being contrasted with the three groups engaged in some form of numerical computation.

2. Treatment Non-Dec was compared with the means of Dec-VM and the Dec-Reg; the group studying non-decimal systems being contrasted with the groups studying decimal systems.

Table 14 shows that the Scheffe Test as applied here indicated that intelligence quotient scores of students in Treatment Dec-VM were significantly inferior to those of the other three treatment groups. This fact alone would have been ample evidence of the necessity to employ analysis of covariance in all posttests.

TABLE 14

COMPARISONS BETWEEN GROUP MEANS USING SCHEFFE TEST FOR CTMM (Pretest) - GRADE FOUR (N=185)

Comparison	Treatment Means				$\sum a_i^2$	d_i	S.E. of d_i	t
	108.67	104.87	93.21	107.46				
(1)vs(2)	1	-1	0	0	2	3.80	2.880	1.319
(1)vs(3)	1	0	-1	0	2	15.46		5.368***
(1)vs(4)	1	0	0	-1	2	-1.21		-.420
(2)vs(3)	0	1	-1	0	2	11.67		4.052***
(2)vs(4)	0	1	0	-1	2	1.21		.420
(3)vs(4)	0	0	1	-1	2	-14.25		-4.948***
(2)vs(1)+ (3)+(4)	-1	3	-1	-1	12	5.27	7.053	.747
(1)vs(3)+ (4)	2	0	-1	-1	6	16.67	4.998	3.335**

The pretest of arithmetic computation was the STAN - Test 1, Form X. Table 15 shows the sums of squares for scores of fourth grade students on this test.

TABLE 15

ANALYSIS OF VARIANCE-STAN TEST 1, FORM X
COMPUTATION (Pretest) GRADE FOUR
(N=185)

Source	SS	df	Mean Square	F-Ratio
Among	485	3	161.547	6.159***
Within	4747	181	26.229	
Total	5232	184		

The significant F-ratio points to the need for Scheffe comparisons between the means. The data for the Scheffe Test is shown in Table 16.

TABLE 16

SCHEFFE COMPARISONS BETWEEN MEANS ON STAN-TEST 1-X
COMPUTATION (Pretest) GRADE FOUR
(N=185)

	Treatment Means						S.E.	
Comparison	13.42	13.23	11.47	15.79	Σa_i^2	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	.19	1.12	.170
(1)vs(3)	1	0	-1	0	2	2.25		2.009
(1)vs(4)	1	0	0	-1	2	-2.37		-2.116
(2)vs(3)	0	1	-1	0	2	2.06		1.839
(2)vs(4)	0	1	0	-1	2	2.56		-2.286*
(3)vs(4)	0	0	1	-1	2	-4.62		-4.125***
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	-.69	2.743	-2.449*
(1)vs(3) +(4)	2	0	-1	-1	6	.12	1.940	1.732

By results shown in Table 16, one may observe the superiority of scores of students in Treatment Dec-Reg over Dec-VM and possibly over Non-Comp; as well as the superiority of the computation groups over the Non-Comp group. This lack of equality of means clearly indicated again a need to attempt statistical equalization through the employment of analysis of covariance in analyzing posttests.

TABLE 17

ANALYSIS OF VARIANCE STAN-TEST 2 X (Pretest)
ARITHMETIC REASONING - GRADE FOUR
(N=185)

Source	SS	df	Mean Square	F-Ratio
Among	354	3	118.078	6.046***
Within	3535	181	19.531	
Total	3889	184		

The significant F-ratio shown in Table 17 indicated that at least one treatment mean was not equal to the others. Therefore the Scheffe Test for comparisons of means was employed. Data for the Scheffe comparisons are shown in Table 18.

TABLE 18

SCHEFFE COMPARISONS BETWEEN MEANS ON STAN-TEST 2-X-ARITHMETIC REASONING - GRADE FOUR (N=185)

		Treatment Means					S.E.	
Comparison	11.56	10.17	8.79	12.50	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	1.39	.964	1.442
(1)vs(3)	1	0	-1	0	2	2.77		2.873^{***}
(1)vs(4)	1	0	0	-1	2	-.92		-.954
(2)vs(3)	0	1	-1	0	2	1.38		1.432
(2)vs(4)	0	1	0	-1	2	-2.33		-2.417#
(3)vs(4)	0	0	1	-1	2	-3.71		-3.849^{***}
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	-2.34	2.361	-.991
(1)vs(3) +(4)	2	0	-1	-1	6	1.83	1.670	1.096

Scheffe comparisons shown in Table 18 indicate a possible ranking of the four treatment groups for grade four in the following order:

Group Number	Treatment	Rank
(4)	Dec-Reg	1
(1)	Non-Dec	2
(2)	Non-Comp	3
(3)	Dec-VM	4

The Dec-Reg seems far above the other three treatments on scores of this pretest of arithmetic reasoning. The latter three groups are much closer together.

Summary of Analysis of Pretest Data

The statistical analysis of the three standardized pretests given students of grade four in this experiment revealed that the four treatment groups could not be considered equal in intelligence, arithmetic computation, or arithmetic reasoning. The Scheffe Test for Comparisons of treatment means, regarded as a conservative test, indicated significant differences on all three pretests.

These results are evidence of the need by the researcher to attempt a statistical equalization of groups on posttests used as criterion measures for this experiment. Therefore on posttests the statistical procedure, analysis of covariance, was employed using selected pretests as covariants.

Hypotheses Concerning Grade Four

For the entire study involving grades four and six, twenty-four hypotheses were formulated. Twelve hypotheses, numbered with odd integers, refer to grade four. These hypotheses are considered one at a time in this chapter.

> Hypothesis 1. There are no significant differences for scores on STAN-Test 1, Form W (Posttest I) - Computation - among groups of fourth grade students receiving the four treatments.

The hypothesis above may be described symbolically as follows:

$H_o: \mu_1 = \mu_2 = \mu_3 = \mu_4$, or H_o: All μ_i are equal.

Analysis of covariance was used to test this hypothesis. If at least one treatment mean varies significantly, the F-ratio will be significant and the alternate hypothesis, H_1: Some u_i are not equal, may be considered to be true with the probability of error not greater than the significance level.

Table 19 shows the analysis of covariance data for STAN-Test 1, Form W (Posttest I), using as covariants the CTMM (Pretest) and the STAN-Test 1, Form X (Pretest).

TABLE 19

ANALYSIS OF COVARIANCE, STAN-TEST 1 COMPUTATION FORM W, (Posttest I) COVARIANTS: CTMM (Pretest) and STAN-TEST 1, FORM X (Posttest) GRADE FOUR (N=185)

Source		SS due to Regression	SS about Regression	df	Mean Square	F-Ratio
Among	1402					
Within	6456	3328	3127	179	17.47	8.656^{***}
Total	7858	4277	3581	182		
Diff. for Testing Among Adjusted Treatment Means			454	3	151.24	

The F-ratio in Table 19 shows significance to the .01 level. Hence, H_o was rejected.

In order to determine the contribution of each covariant to the analysis, the coefficients for covariants, which may be used in the computation of a regression equation, were examined. Significant t-values would indicate that the regression coefficients are non-zero and that each covariant had contributed to the analysis of this test.

Table 20 lists the pooled within-treatment regression coefficients for each covariant and the regression coefficients for the total experimental population as well as for each standard error and t-value.

TABLE 20

COEFFICIENTS FOR COVARIANTS FOR DATA OF TABLE 19

Source	CTMM (Pretest) Coeff.	S.E.	t	STAN-Test 1 Coeff.	Form X (Pretest) S.E.	t
Within	.0968	.0259	3.7315^{***}	.7011	.0669	10.4851^{***}

TABLE 20 -- Continued

Source	CTMM (Pretest) Coeff.	S.E.	t	STAN-Test 1 Coeff.	Form X (Pretest) S.E.	t
Total	.0938	.0257	3.6496***	.7584	.0693	10.9478***

Both the CTMM and the STAN-Test 1 (Pretest) contributed significantly to this analysis.

Adjusted treatment means, their standard errors, and adjusted variances are shown in Table 21.

TABLE 21

ADJUSTED TREATMENT MEANS AND STANDARD ERRORS
FOR DATA OF TABLE 19-STAN-TEST 1
COMPUTATION-FORM W (Posttest I)

Treatment	N	Mean	Adj. Mean	S.E. Adj. Mean	Adj. Var.	$\frac{\text{Adj. Var. max}}{\text{Adj. Var. min}}$
Non-Dec	43	16.72	16.28	.6503	18.18	1.15
Non-Comp	52	14.29	14.35	.5809	17.55	
Dec-VM	42	13.76	16.39	.6923	20.13	
Dec-Reg	48	20.67	18.68	.6203	18.47	

Homogeneity of adjusted variances was indicated by the non-significant max-F ratio in the right-hand column of Table 21.

Ferguson (1959) indicated that an assumption of linearity could be made for most tests in psychology and education. This assumption was borne out by the similarity of pooled within-class regression coefficients and the total group regression coefficients in Table 20. In both cases, differences between regression coefficients were far less than the sum of their standard errors.

In this case, linearity of the overall regression was assumed because use of two covariants did not permit separation of sums of squares data into components needed for this type of statistical analysis.

Table 22 presents data for the Scheffe comparisons between the adjusted treatment means on the posttest of arithmetic computation.

TABLE 22

SCHEFFE COMPARISONS BETWEEN ADJUSTED MEANS - STAN-TEST 1, FORM W (Posttest I) GRADE FOUR (N=185)

Comparison	Adj. Treatment Means 16.28	14.35	16.39	18.68	$\sum a_i^2$	d_i	S.E. d_i	t
(1)vs(2)	1	-1	0	0	2	1.93	.913	2.114
(1)vs(3)	1	0	-1	0	2	-.11		-.120
(1)vs(4)	1	0	0	-1	2	2.40		-2.629*
(2)vs(3)	0	1	-1	0	2	-2.04		-2.234
(2)vs(4)	0	1	0	-1	2	-4.33		-4.743***
(3)vs(4)	0	0	1	-1	2	-2.29		-2.508*
(2)vs(1)+ (3)+(4)	-1	3	-1	-1	12	-8.30	2.236	-3.713***
(1)vs(3)+ (4)	2	0	-1	-1	6	2.51	1.581	1.588

The ranking of treatment groups which may be assumed from the Scheffe comparisons is as follows:

Group Number	Treatment	Rank
(4)	Dec-Reg	1
(1)	Non-Dec	2.5
(3)	Dec-VM	2.5
(2)	Non-Comp	4

Although various computational treatments differed, any computational treatment produced superior scores to the Non-Comp group. This was indicated on the posttest of arithmetic computation.

On the basis of computation scores, fourth grade students appeared to have "suffered" from lack of number work during their five week study of geometry.

Hypothesis 3. There are no significant differences for scores on STAN-Test 2, Form W (Posttest I) - Arithmetic Reasoning - among groups of fourth grade students receiving the four treatments.

Derived data for the analysis of covariance for this test of arithmetic reasoning are shown in Table 23.

TABLE 23

ANALYSIS OF COVARIANCE: STAN-TEST 2W (Posttest I)
ARITHMETIC REASONING. COVARIANTS:
CTMM (Pretest) AND STAN-TEST 2 X (Pretest) GRADE FOUR
(N=185)

Source	SS	SS due to Regression	SS About Regression	df	Mean Square	F-Ratio
Among	698					
Within	5038	2886	2152	179	12.02	1.597
Total	5736	3526	2210	182		
Diff. for Testing Among Adjusted Treatment Means			58	3	19.19	

Because of the non-significant F-ratio in the analysis of covariance for scores of STAN-Test 2, Form W (Posttest I) as shown in Table 23, the adjusted treatment means were considered to be equal and $H_o:\mu_i$ are equal was not rejected.

Table 24 shows the pooled within-treatment regression coefficients and the overall regression for both covariants. Computed standard errors and t-scores indicated non-zero regression coefficients in all cases.

TABLE 24

COEFFICIENTS FOR COVARIANTS FOR DATA IN TABLE 23

Source	CTMM Coeff.	(Pretest) S.E.	t	STAN-Test 2X Coeff.	S.E.	(Pretest) t
Within	.1405	.0225	6.255***	.6192	.0671	9.2292***
Total	.1390	.0212	6.554***	.6452	.0663	9.7350***

The minimal differences between the pooled within-treatment regression coefficients and the overall regression coefficients may help substantiate the linearity of the regression line. Linearity of regression was assumed because supporting statistical data was unavailable.

Table 25 shows adjusted treatment means, standard errors, adjusted variances for each treatment, and homogeneity of adjusted variances.

TABLE 25

ADJUSTED TREATMENT MEANS, STANDARD ERRORS, AND ADJUSTED VARIANCES FOR DATA IN TABLE 23

Treatment	N	Mean	Adj. Mean	S.E. Adj.Mean	Adj. Var.	$\frac{\text{Adj.Var.max}}{\text{Adj.Var.min}}$
Non-Dec	43	14.21	13.04	.5373	12.41	1.13
Non-Comp	52	12.54	12.76	.4842	12.19	
Dec-V	42	10.50	13.22	.5736	13.82	
Dec-Reg	48	15.81	14.23	.5110	12.53	

This posttest of arithmetic reasoning yielded no significant differences among treatment means adjusted for intelligence quotient and arithmetic achievement.

Hypothesis 5. There are no significant differences for scores on STAN-Test 1, Form X (Posttest II) - Computation - among groups of fourth grade students receiving the four treatments.

Data for analysis of the Arithmetic Test of Computation, STAN-Test IX (Posttest II), using as covariants CTMM (Pretest), STAN-Test IX (Pretest), and STAN-Test 2W (Posttest I) are shown in Table 26.

TABLE 26

ANALYSIS OF COVARIANCE STAN-TEST 1 - COMPUTATION - FORM X, (Posttest II). COVARIANTS: CTMM (Pretest), STAN-TEST IX (Pretest), AND STAN-TEST 1W (Posttest I) (N=185)

Source	SS	SS Due to Regression	SS About Regression	df	Mean Square	F-Ratio
Among	847					
Within	6031	3383	2648	178	14.87	1.891
Total	6878	4145	2732	181		
Diff. for Testing Among Adjusted Treatment Means			84	3	28.13	

The non-significant F-ratio of means squares on the retention test on arithmetic computation may be seen in Table 26. Therefore, the hypothesis of equal treatment means was not rejected.

Retention tests were given in late May and early June, about twelve weeks after the CTMM (Pretest), the possibility existed, therefore, that the intelligence test scores which formed one covariant might not be relevant for this retention test. Accordingly, a similar analysis of covariance was performed for this test using instead the CTMM (Posttest) scores.

These scores had resulted from a second rendition of the test early in June and might be assumed to represent a truer evaluation at the time of the May-June testing; although these scores were very likely influenced by the experiment itself.

Table 27 shows the analysis of covariance using the CTMM (Posttest) as one covariant.

TABLE 27

ANALYSIS OF COVARIANCE STAN-TEST 1 - ARITHMETIC
COMPUTATION - FORM X (Posttest 1)
COVARIANTS: CTMM (Posttest), STAN-TEST 1X
(Pretest) and STAN-TEST 1W (Posttest)
(N=185)

Source	SS	SS Due to Regression	SS About Regression	df	Mean Square	F-Ratio
Among	867					
Within	6031	3059	2972	178	16.70	1.037
Total	6878	3853	3024	181		
Diff. for Testing Among Adjusted Treatment Means			52	3	17.31	

The F-ratio in Table 27, is smaller than the F-ratio in Table 26, indicating less possibility of non-rejection of the null hypothesis.

Table 28 shows coefficients for covariants for the analysis using CTMM (Pretest); Table 29, similarly, for the analysis using CTMM (Posttest).

TABLE 28

COEFFICIENTS FOR COVARIANTS FOR DATA OF TABLE 26

	CTMM (Pretest)			STAN-Test IX (Pretest)			STAN-Test 1W (Posttest)		
Source	Coeff.	S.E.	t	Coeff.	S.E.	t	Coeff.	S.E.	t
Within	.0818	.0249	3.2931***	.2740	.0784	3.4948***	.4324	.0690	6.2695***
Total	.1007	.0233	4.3168***	.2768	.0781	3.5431***	.4032	.0649	6.2104***

TABLE 29

COEFFICIENTS FOR COVARIANTS FOR DATA OF TABLE 27

	CTMM (Pretest)			STAN-Test IX (Pretest)			STAN-Test IW (Posttest)		
Source	Coeff.	S.E.	t	Coeff.	S.E.	t	Coeff.	S.E.	t
Within	.1211	.0252	4.8094***	.4809	.0738	6.5181***	.1705	.0913	1.8677
Total	.1259	.0230	5.4818***	.4727	.0729	6.4805***	.1853	.0904	2.0488

It is interesting to observe from Table 29 that the regression coefficients (pooled within-treatment and also overall) for the STAN-Test 1, Form W (Posttest I) are not significantly different from zero to be noteworthy. The test, used as covariant, contributed very little to the prediction of scores on the retention test and to the overall regression equation.

Whatever differences may have been evidenced in computational ability among students in the four treatment groups on the posttest immediately following the teaching period appeared to have disappeared when groups were retested following seven weeks of the usual arithmetic program.

Tables 30 and 31 are provided to show data for homogeneity of variance (Hartley Max-F Test) for the analysis of Tables 26 and 27.

TABLE 30

ADJUSTED TREATMENT MEANS, STANDARD ERRORS, AND ADJUSTED VARIANCES FOR DATA OF TABLE 26

Treatment	N	Mean	Adj. Mean	S.E. Adj.Mean	Adj. Var.	Adj.Var.max / Adj.Var.min
Non-Dec	43	18.30	17.77	.6000	15.48	1.11
Non-Comp	52	16.19	17.08	.5541	15.97	
Dec-VM	42	13.31	15.94	.6388	17.14	
Dec-Reg	48	18.94	16.15	.5938	16.92	

TABLE 31

ADJUSTED TREATMENT MEANS, STANDARD ERRORS, AND ADJUSTED VARIANCES FOR DATA IN TABLE 27

Treatment	N	Mean	Adj. Mean	S.E. Adj.Mean	Adj. Var.	Adj.Var.max / Adj.Var.min
Non-Dec	43	18.30	17.64	.6331	17.24	1.20
Non-Comp	52	16.19	16.18	.5730	17.07	
Dec-VM	42	13.30	16.47	.6981	20.47	
Dec-Reg	48	18.94	16.48	.6111	17.93	

The F-ratios in Tables 30 and 31 indicate that adjusted variances are homogeneous.

Hypothesis 7. There are no significant differences for scores on STAN-Test 2, Form X (Posttest II) - Arithmetic Reasoning - among groups of fourth grade students receiving the four treatments.

Retention test scores on the test of arithmetic reasoning, STAN-Test 2, Form X (Posttest II) are analyzed in Table 32.

TABLE 32

ANALYSIS OF COVARIANCE OF STAN-TEST 2X (Posttest II). COVARIANT: CTMM (Pretest) STAN-TEST 2X (Pretest), AND STAN-TEST 2W (Posttest I) (N=185)

Source	SS	SS Due to Regression	SS About Regression	df	Square	F-Ratio
Among	1077					
Within	5010	3297	1714	178	9.63	2.795^{***}
Total	6087	4292	1795	181		
Diff for Testing Among Adjusted Treatment Means			81	3	26.91	

Table 32 shows the covariants for analysis to be the CTMM (Pretest), STAN-Test 2, Form X (Pretest), and STAN-Test 2, Form W (Posttest I). The F-ratio was significant to the .05 level. In this case the null hypothesis H_o: $\mu_i = 0$ was rejected.

When scores for the same test of arithmetic reasoning were analyzed with the CTMM (Posttest) as covariant instead of the CTMM (Pretest) (Table 33), for reasons detailed in the discussion of Hypothesis 5, the F-ratio was significant approximately to the .10 level.

TABLE 33

ANALYSIS OF COVARIANCE STAN-TEST 2X (Posttest II) COVARIANT: CTMM (Posttest), STAN-Test 2X (Pretest), and STAN-TEST 2W (Posttest I) (N=185)

Source	SS	SS Due to Regression	SS About Regression	df	Mean Square	F-Ratio
Among	1077					
Within	5010	3326	1684	178	9.46	2.117
Total	6087	4343	1744	181		
Diff. for Testing Among Adjusted Treatment Means			60	3	20.04	

This condition led to the non-rejection of the hypothesis and the four treatment means would have been considered equal.

For this discussion, the treatment means shall be considered to be unequal, though not markedly different

To ascertain where the slight differences might be, the Scheffe test of comparison of means was employed. Data for this test is shown in Table 34.

TABLE 34

SCHEFFE COMPARISONS BETWEEN ADJUSTED MEANS FOR DATA OF TABLE 32

	Adj. Treatment Means						S.E.	
Comparison	14.55	13.59	12.58	14.22	$\sum a_i^2$	d_i	d_i	t
(1)vs(2)	1	-1	0	0	2	.96	6.77	1.48
(1)vs(3)	1	0	-1	0	2	1.97		2.910**
(1)vs(4)	1	0	0	-1	2	.33		.487
(2)vs(3)	0	1	-1	0	2	1.01		1.492
(2)vs(4)	0	1	0	-1	2	.63		-.931
(3)vs(4)	0	0	1	-1	2	-1.64		-2.422#
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	-.58	1.658	-.350
(1)vs(3) +(4)	2	0	-1	-1	6	2.30	1.173	1.961

The ranking which may be assumed for the four treatment groups on this retention test of arithmetic reasoning was as follows:

Group Number	Treatment	Rank
(1)	Non-Dec	1.5
(4)	Dec-Reg	1.5
(2)	Non-Comp	3
(3)	Dec-VM	4

It should be noted that the differences are small, especially between the last two means.

Tables 35 and 36 indicate regression coefficients for the three covariants used in the analysis of the retention test of arithmetic reasoning.

TABLE 35

COEFFICIENTS FOR COVARIANTS FOR DATA OF TABLE 32

	CTMM (Pretest)			STAN-Test 2X (Pretest)			STAN-Test 2W (Posttest I)		
Source	Coeff.	S.E.	t	Coeff.	S.E.	t	Coeff.	S.E.	t
Within	.0614	.0222	2.7695***	.2923	.0729	4.0078***	.5133	.0669	7.6738***
Total	.0780	.0213	3.6610***	.3009	.0739	4.0731***	.5211	.0670	7.7794***

TABLE 36

COEFFICIENTS FOR COVARIANTS FOR DATA OF TABLE 33

	CTMM (Posttest)			STAN-Test 2X (Pretest)			STAN-Test 2W (Posttest I)		
Source	Coeff.	S.E.	t	Coeff.	S.E.	t	Coeff.	S.E.	t
Within	.0688	.0208	3.3062***	.2797	.0726	3.8504***	.4941	.0669	7.3809***
Total	.0838	.0192	4.3587***	.2845	.0732	3.8876***	.4951	.0668	7.4098***

The t-values for the regression coefficients in Tables 35 and 36 indicated that in both cases, all coefficients were non-zero to the .01 level.

Data for the test of homogeneity of adjusted variances is shown in Tables 37 and 38. In both cases, homogeneity was clearly established and use of analysis of covariance was upheld.

TABLE 37

ADJUSTED TREATMENT MEANS, STANDARD ERRORS AND ADJUSTED VARIANCES FOR DATA OF TABLE 32

Treatment	N	Mean	Adj. Mean	S.E. Adj.Mean	Adj. Var.	Adj.Var.max / Adj.Var.min
Non-Dec	43	15.54	14.55	.4812	9.96	1.13
Non-Comp	52	13.08	13.59	.4349	9.84	
Dec-VM	42	9.91	12.58	.5134	11.07	
Dec-Reg	48	16.23	14.22	.4615	10.55	

TABLE 38

ADJUSTED TREATMENT MEANS, STANDARD ERRORS, AND ADJUSTED VARIANCES FOR DATA OF TABLE 33

Treatment	N	Mean	Adj. Mean	S.E. Adj.Mean	Adj. Var.	Adj.Var.max / Adj.Var.min
Non-Dec	43	15.54	14.56	.4749	9.70	1.20
Non-Comp	52	13.08	13.50	.4337	9.78	
Dec-VM	42	9.91	12.53	.5256	11.60	
Dec-Reg	48	16.22	14.09	.4588	10.10	

Hypothesis 9. There are no significant differences for CTMM (Posttest II) scores when groups have been matched according to CTMM (Pretest) among fourth grade students.

Data from the two renditions of the CTMM (Pretest) and (Posttest) seemed to produce slightly different results in connection with analyses of achievement tests; and because the CTMM and STAN achievement tests may be measuring similar factors, an analysis of covariance for CTMM (Posttest) was performed with the covariant CTMM (Pretest)

Table 39 presents an expanded table of sums of squares. Additional data is presented both for the within-treatment sums of squares and the sums of squares and cross products for the covariant because in this case, since there was only one covariant, the data is available. The F-ratio indicates a rejection of the null hypothesis:
$H_o: \mu_1 = \mu_2 = \mu_3 = \mu_4$.

TABLE 39

ANALYSIS OF CTMM (Posttest)
COVARIANT: CTMM (Pretest) GRADE FOUR
(N=185)

Source	Σx^2	Σxy	Σy^2	df	$\Sigma y^2 - \frac{(\Sigma xy)^2}{\Sigma x^2}$	MS	F	df (adj)	MS^1	
Within										
Non-Dec.	6689	7125	10230		2641					
Non-Comp.	9572	9004	10491		2021					
Dec-VM	6763	6914	7972		904					
Dec-Reg.	8518	7864	9072		1812					
					$S_1=7378$					
Between Groups	6429	8449	11352	3	$S_3=248$	3783	18.135***			
Within Groups	31543	30908	37765	181	$S_p=S_1+S_2=$ 7480			180	41.56	6.775***
Total	37972	39357	49117	184	$S_T=8324$	209		183		
Diff. for Testing Among Adjusted Treatment Means					$S_3+S_4=$ 844			3	281.54	

Note: $S_2=S_p-S_1$; $S_4=S_T-S_p-S_3$

A separate listing of variance used in testing the assumptions is found in Table 41.

TABLE 40

PARTITIONS OF VARIANCE FOR TESTS ON ASSUMPTIONS UNDERLYING ANALYSIS OF COVARIANCE OF CTMM (Posttest)

S_i	Value	df	Interpretation	Symbols
S_1	7378	177	$\sum(n_i-1)-r-(k-1)$	r = number of regression coefficients;
S_2	102	3	k-1	k = number of treatments;
S_3	248	2	k-2	n = number of scores in one treatment
S_4	596	1	1	
S_T	8324	183	$\sum(n_i-1)-r+(k-1)$	

A study of Table 41 discloses that the one assumption that the between-treatment coefficient was equal to the within-treatment regression was violated. This was test 2(c) of Table 41. This amounts to saying that the regression of Y on X is heterogeneous and that there is a "treatment" effect in which the relative effectiveness of the treatments differ for different values of the covariant.

As explained by Lindquist (1953), there may be some values of the covariant for which the treatments are equally effective and others in which one treatment is superior to another. In this instance we may be showing that test reliability for the CTMM depends on the original score.

TABLE 41

TESTS ON ASSUMPTIONS UNDERLYING ANALYSIS OF COVARIANCE FOR CTMM (Posttest)

Description	Test	F-Ratio
(1) Difference in Means	$F = \dfrac{\frac{S_3+S_4}{k-1}}{\frac{S_1+S_2}{\sum(n_i-1)-r}}$	6.775***
(2) Can one regression line be used for all observations, i.e., is the overall regression linear? If significant, use (a), then (b), and then (c)	$F = \dfrac{\frac{S_2+S_3+S_4}{(k-1)}}{\frac{S_1}{\sum(n_i-1)-r-(k-1)}}$	.379
(a) Are the slopes of regression lines within treatment groups the same?	$F = \dfrac{\frac{S_2}{k-1}}{\frac{S_1}{\sum(n_i-1)-r-(k-1)}}$	.081
(b) Is the between treatment regression linear?	$F = \dfrac{\frac{S_3}{-\ k-2}}{\frac{S_1+S_2}{\sum(n_i-1)-r}}$	2.983
(c) If slopes are the same and regression for means is linear, are between treatment regression coefficients the same as within treatment regression coefficients?	$\dfrac{\frac{S_4}{1}}{\frac{S_1+S_2}{\sum(n_i-1)-r}}$	14.34***

Test (2) of Table 41 showed that the approximate overall regression line was linear. Test 2(a) showed that the regression coefficients within-treatments were equal. More important, Test 2(b) showed that the between-treatment regression was linear, although the F-ratio in this case was nearing significance.

Since the major assumptions were upheld, the analysis of covariance was assumed to be appropriate.

The within-treatment coefficient, its standard error of estimate, and t-value are .9799, .0363, and 26.9963***, respectively. The total regression coefficient, its standard error, and its t-value are 1.0365, .0346, and 29.9457***, respectively. The non-zero nature of the regression coefficient was clearly indicated here.

Table 42 shows homogeneity of adjusted variances.

TABLE 42

ADJUSTED TREATMENT MEANS, STANDARD ERRORS, AND ADJUSTED VARIANCES FOR DATA OF TABLE 39

Treatment	N	Mean	Adj. Mean	S.E. Adj.Mean	Adj. Var.	$\frac{\text{Adj.Var.max}}{\text{Adj.Var.min}}$
Non-Dec	43	111.37	106.57	.9990	42.91	1.15
Non-Comp	52	108.69	107.63	.8948	41.63	
Dec-VM	42	92.62	102.97	1.0660	47.73	
Dec-Reg	48	112.96	109.36	.9400	42.41	

Table 43 displays data used in the Scheffe comparisons of treatment means.

TABLE 43

SCHEFFE COMPARISONS BETWEEN ADJUSTED MEANS
FOR DATA OF TABLE 39

	Adj. Treatment Means							
Comparison	106.57	107.63	102.97	109.35	$\sum a_i^2$	d_i	S.E. of d_i	t
(1)vs(2)	1	-1	0	0	2	-1.06	1.41	.752
(1)vs(3)	1	0	-1	0	2	3.60		2.553*
(1)vs(4)	1	0	0	-1	2	-2.78		-1.972
(2)vs(3)	0	1	-1	0	2	4.66		3.305**
(2)vs(4)	0	1	0	-1	2	-1.72		-1.220
(3)vs(4)	0	0	1	-1	2	-6.38		-4.525***
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	4.00		1.159
(1)vs(3) +(4)	2	0	-1	-1	6	.82		.336

The data of Table 43 disclosed the following ranking for the four treatment groups:

Group Number	Treatment	Rank
(4)	Dec-Reg	2
(2)	Non-Comp	2
(1)	Non-Dec	2
(3)	Dec-VM	4

The Dec-VM group also started with a lower group mean, from which one may conjecture that improvement in scores may be a function of a starting score. The possibility exists, however, that the particular treatment had an adverse effect on the intelligence quotient scores. Because this comparison of the CTMM is based on group mean scores, one may only conjecture what this change means with respect to intelligence quotient and its relation to the standard error of the test itself.

The relative positions of the group means as indicated in Table 43 suggests that the intelligence quotients of the Dec-VM group did not keep pace with that of the other three groups.

> Hypothesis 11. There are no differences for difference scores between the Pretest and Posttest I STAN-Test 1 scores among groups of fourth grade students receiving the four treatments.

Differences between scores on the STAN-Test 1, Form X (Pretest) and the STAN-Testl, Form W (Posttest) were analyzed by means of analysis of variance. Table 34 showed the partitioning of the variance and a highly significant F-ratio indicating rejection of the null hypothesis of equal means.

Appendix H includes a table showing the test of homogeneity of variance for the analysis of variance performed in the testing of Hypothesis 11-20. All data but the scores used in connection with Hypothesis 18 are suited to analysis of variance on this basis.

TABLE 44

ANALYSIS OF VARIANCE DIFFERENCE SCORES
STAN-TEST 1 (Posttest I - Pretest)
ARITHMETIC COMPUTATION GRADE FOUR
(N=185)

Source	SS	df	Mean Square	F-Ratio
Among	374	3	124.84	6.366***
Within	3549	181	19.61	
Total	3923	184		

TABLE 45

SCHEFFE COMPARISONS BETWEEN MEANS ON DIFFERENCE SCORES-STAN-TEST 1 (Posttest I - Pretest) ARITHMETIC COMPUTATION

	Treatment Means						S.E.	
Comparisons	3.3023	1.0577	2.5952	4.8750	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	2.2446	.961	2.336#
(1)vs(3)	1	0	-1	0	2	.7071		.736
(1)vs(4)	1	0	0	-1	2	-1.5727		-1.636
(2)vs(3)	0	1	-1	0	2	-1.5375		-1.600
(2)vs(4)	0	1	0	-1	2	-3.8173		-3.972***
(3)vs(4)	0	0	1	-1	2	-2.2798		-2.372#
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	-7.5994	2.353	-3.230**
(1)vs(3) +(4)	2	0	-1	-1	6	.8656	1.664	.520

Table 45, the Scheffe Test data, indicated the following approximate ranking of treatment groups on these difference scores:

Group Number	Treatment	Rank
(4)	Dec-Reg	1
(3)	Dec-VM	2
(1)	Non-Dec	3
(2)	Non-Comp	4

It may be recalled that in the ranking on the pretest, Dec-Reg was first and Dec-VM was last. Gains in scores were clearly in favor of the Dec-Reg treatment group. The least progress in computation was reported for the Non-Comp (Geometry) treatment group. This was borne out in the Scheffe comparison showing the average of the computation group to surpass significantly the Non-Comp group.

Hypothesis 13. There are no differences for difference scores between the Pretest and Posttest I STAN-Test 2 among groups of fourth grade students receiving the four treatments.

The non-significant F-ratio resulting from analysis of variance of the score differences between STAN-Test 2, Form X (Pretest) and the STAN-Test 2, Form W (Posttest I) indicated that treatment means may be considered to be equal. (Table 46)

TABLE 46

ANALYSIS OF VARIANCE DIFFERENCE SCORES
STAN-TEST 2 (Posttest I - Pretest)
ARITHMETIC REASONING
(N=185)

Source	SS	df	Mean Square	F-Ratio
Among	59	3	19.79	1.313
Within	2729	181	15.08	
Total	2788	184		

The null hypothesis was not rejected and the difference score means on this test of arithmetic reasoning were considered equal.

Hypothesis 15. There are no significant differences for difference scores between the Posttest I and Posttest II STAN-Test 1 scores among the fourth grade students receiving the four treatments.

Analysis of variance of the difference scores on the test of computation between the renditions Posttest I and Posttest II showed a significant F-ratio and indicated rejection of the hypothesis of equal means. (Table 47)

TABLE 47

ANALYSIS OF VARIANCE DIFFERENCE SCORES
STAN-TEST (Posttest II - Posttest I)
(N=185)

Source	SS	df	Mean Square	F-Ratio
Among	425	3	141.79	6.904**
Within	3717	181	20 54	
Total	4142	184		

Analysis of mean differences by the Scheffe Test shown in Table 48.

TABLE 48

SCHEFFE COMPARISONS BETWEEN MEANS ON DIFFERENCE SCORES
STAN-TEST 1 - COMPUTATION (Posttest II - Posttest I)
GRADE FOUR (N=185)

	Treatment Means						S.E.	
Comparison	1.5814	1.9038	-.4524	-1.7292	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	-.3224	.989	-.326
(1)vs(3)	1	0	-1	0	2	2.0338		2.056
(1)vs(4)	1	0	0	-1	2	3.3106		3.347**
(2)vs(3)	0	1	-1	0	2	2.3562		2.382#
(2)vs(4)	0	1	0	-1	2	3.6330		3.673***
(3)vs(4)	0	0	1	-1	2	2.1816		2.206
(2)vs(1)+ (3)+(4)	-1	3	-1	-1	12	6.3116	2.472	2.553*
(1)vs(3)+ (4)	2	0	-1	-1	6	5.3444	1.713	3.119**

It may be observed that certain "losses" shown in the teaching period of the experiment seem to have been overcome during the post-teaching period.

> Hypothesis 17. There are no significant differences for difference scores between the Posttest I and Posttest II STAN-Test 2 scores among the fourth grade students receiving the four treatments.

Analysis of variance indicated a non-significant F-ratio for the difference scores on the test of arithmetic reasoning between Posttest I and Posttest II. (Table 49)

TABLE 49

ANALYSIS OF VARIANCE DIFFERENCE SCORES
STAN-TEST 2 (Posttest II-Posttest I)
ARITHMETIC REASONING

Source	SS	df	Mean Square	F-Ratio
Among	79	3	26.42	2.148
Within	2226	181	12.38	
Total	2305	184		

The null hypothesis of equal means was not rejected.

> Hypothesis 19. There are no significant differences for scores on the sub-portion of STAN-Test 2 directly testing the concept of place value and numeration among fourth grade students receiving the four treatments.

In the test of arithmetic reasoning, STAN-Test 2, several questions required knowledge of concepts of place value and numberation. A group of seven questions was selected from Form X and matched with a set from Form W. An example of the matching process may be found in Appendix J. This set of matched questions was referred to by the name, Place Value Subtest in several tables presented earlier.

Each child's responses to each of these seven questions was listed on his IBM data card. This listing enabled computation of reliabilities for this test by the Kuder-Richardson 20 index of test reliability. (Table 8)

Analysis of covariance, followed by the Scheffe comparison of means test was used on all scores. Tables 50 and 51 display data for the Place Value Pretest; Tables 52 and 53 for the Place Value Posttest I; and Tables 54 and 55 for the Place Value Posttest II.

TABLE 50

ANALYSIS OF VARIANCE: PLACE VALUE SUBTEST (Pretest)
GRADE FOUR (N=185)

Source	SS	df	Mean Square	F-Ratio
Among	27.67	3	9.223	4.273***
Within	390.71	181	2.159	
Total	418.38	184		

TABLE 51

SCHEFFE COMPARISONS BETWEEN MEANS ON PLACE VALUE SUBTEST (Pretest) GRADE FOUR

Comparison	Treatment Means 3.2791	3.2115	2.5476	3.6458	$\sum a_i^2$	d_i	S.E. of d_i	t
(1)vs(2)	1	-1	0	0	2	.0678	.320	.212
(1)vs(3)	1	0	-1	0	2	.7315		2.286#
(1)vs(4)	1	0	0	-1	2	-.3667		-1.146
(2)vs(3)	0	-1	-1	0	2	.6638		2.074
(2)vs(4)	0	1	0	-1	2	-.4343		1.3572
(3)vs(4)	0	0	1	-1	2	-1.0982		-3.432^{***}
(2)vs(1)+(3)+(4)	-1	3	-1	-1	12	.1620	.784	2.066
(1)vs(3)+(4)	2	0	-1	-1	6	.3648	.554	.658

TABLE 52

ANALYSIS OF VARIANCE: PLACE VALUE SUBTEST (Posttest I) (N=185)

Source	SS	df	Mean Square	F-Ratio
Among	37.74	3	12.579	3.702^{**}
Within	615.00	181	3.398	
Total	652.74	184		

TABLE 53

SCHEFFE COMPARISON BETWEEN MEANS ON PLACE VALUE SUBTEST (Posttest I)

	Treatment Means						S.E.	
Comparison	4.2558	4.0000	3.1667	4.3542	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	.2558	.402	.636
(1)vs(3)	1	0	-1	0	2	1.0891		2.709*
(1)vs(4)	1	0	0	-1	2	-.0984		-.245
(2)vs(3)	0	1	-1	0	2	.8333		2.048
(2)vs(4)	0	1	0	-1	2	-.3542		-.881
(3)vs(4)	0	0	1	-1	2	-1.1875		-2.954**
(2)vs(1)+ (3)+(4)	-1	3	-1	-1	12	.2233	.984	.227
(1)vs(3)+ (4)	2	0	-1	-1	6	.9907	.696	1.423

TABLE 54

ANALYSIS OF VARIANCE: PLACE VALUE SUBTEST (Posttest II) GRADE FOUR (N=185)

Source	SS	df	Mean Square	F-Ratio
Among	19.15	3	6.388	3.249**
Within	355.73	181	1.965	
Total	374.89	184		

TABLE 55

SCHEFFE COMPARISON BETWEEN MEANS ON PLACE VALUE SUBTEST (Posttest II) GRADE FOUR

	Treatment Means						S.E.	
Comparison	3.6744	3.5769	3.2381	4.1458	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	.0975	.306	.319
(1)vs(3)	1	0	-1	0	2	.4363		1.426
(1)vs(4)	1	0	0	-1	2	-.4714		-1.541
(2)vs(3)	0	1	-1	0	2	.3388		1.107
(2)vs(4)	0	1	0	-1	2	-.5689		-1.859
(3)vs(4)	0	0	1	-1	2	-.9077		-2.966**
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	-.3276	.749	-.437
(1)vs(3) +(4)	2	0	-1	-1	6	-.0351	.530	-.066

In all three analyses of various significant F-ratios (.05 level), the null hypothesis of equal means was rejected and the Scheffe comparison of means test was employed.

Results on the three renditions of the Place Value Subtest pointed to a consistent ranking of the groups on these scores as follows:

Group Number	Treatment	Rank
(4)	Dec-Reg	1
(1)	Non-Dec	2.5
(2)	Non-Comp	2.5
(3)	Dec-VM	4

It should be observed that these rankings are approximate and that the differences between one treatment group and the next are not the same for each test.

For its length, this place value subtest proved to be fairly reliable. Its value to this experiment lay in its providing further evidence concerning the learning of place value concepts, one of the objectives of the teaching of non-decimal systems, by students in the various treatment groups.

> Hypothesis 21. There are no significant correlations for fourth grade students separated according to sex and treatment among scores for intelligence, teacher judgment of arithmetic and reading ability, arithmetic computation, arithmetic reasoning, non-decimal numeration, and geometry.

Intercorrelations computed using the Pearson Product-Moment formula were calculated for the variables described in Appendix E for various groups of fourth grade students. Table 56 displays the correlation coefficients for the fourth grade boys in the lower left half of the table and for the fourth grade girls in the upper right half. The variables selected for study are age, teacher reading and arithmetic estimates, the pretests, and the posttests of non-decimal numeration, geometry, and place value.

The hypothesis being tested for each pair of variables may be stated symbolically, $H_0: \rho_{ij} = 0$. Not all correlations displayed in Table 56 are considered significant (Appendix F). For this study, only correlations significant to the .01 level are considered noteworthy; these are shown underscored in the tables which follow.

Of the 65 correlations shown for each group in Table 56, 22 are significant for the boys and 25 for the girls.

TABLE 56

INTERCORRELATIONS OF SCORES OF GRADE FOUR STUDENTS

Boys N=101[1] Girls N=107[2]

Variable[3]	(9)	(11)	(12)	(13)	(14)	(15)	(16)	(18)	(19)	(24)	(26)	(28)
(9) Age		-06	-01	32	30	28	-12	-07	-09	05	-32	05
(11) Rdg. Estimate (Tr)	-14		67	-06	-07	02	70	32	45	21	06	47
(12) Arith Estimate (Tr)	-15	52		00	-02	06	57	36	46	18	-01	46
(13) Visual Pretest	60	-09	07		85	77	04	04	07	10	02	16
(14) Auditory Pretest	51	08	-03	77		81	-03	-07	02	08	10	09
(15) Motor Coord. Pretest	43	14	14	73	65		02	-02	06	05	17	GIRLS 03
(16) CTMM Pretest	-22	65	43	-08	-05	11		44	58	30	05	55
(18) Stan-Arith.Comp. Pretest	-07	43	45	-02	-01	06	44		67	09	08	47
(19) Stan-Arith.Reas. Pretest	BOYS 01	62	36	-04	03	12	59	52		24	-12	28
(24) Non-Dec. Posttest I	-09	30	04	-06	05	03	21	20	26			-06
(26) Geometry Postest I	-12	01	06	04	07	13	13	-05	04			
(28) Place-Value Postest I	-12	54	37	00	01	11	53	49		16	17	

1. With an N=101, r must equal .25 to be significant at the .01 level.
2. With an N=107, r must equal .24 to be significant at the .01 level.
3. Numbered as in Appendix E

The three pairs of correlation coefficients for boys and girls which appeared to differ were further examined by testing for the significance of the difference of the correlation coefficient (Garrett, 1958, p. 241). The correlations for boys and girls groups on the pairs of variables (11), (24); (9), (26); and (16), (24) did not show differences significant even to the .05 level. A difference of approximately .25 in the r values would have been necessary in order to have done so. For all pairs of variables, the correlations of fourth grade boys did not differ significantly from those for girls.

The non-significance of correlations for the pretests of vision, hearing, and motor coordination with the standardized tests of intelligence and arithmetic may be seen in Table 56. The geometry test showed no correlations with standardized test scores. The non-decimal test correlated significantly with the teachers' estimates of reading and the arithmetic reasoning scores. The place value test showed significant correlations with both intelligence and arithmetic tests.

Table 57 shows 54 intercorrelations for the Non-Dec group of which 26 are significant to the .01 level and 54 for the Non-Comp group of which 21 are significant. The findings on Table 57 should be considered along with those displayed on Table 58.

Table 58 shows 44 correlations for the Dec-VM group of which 10 are significant to the .01 level and 44 correlations for the Dec-Reg group of which 15 are significant.

In order to show any differences among correlation coefficients of these four treatment groups a difference of approximately .12 was necessary (Garrett, p. 242). Correlations for the four groups between the reading estimate and computation were .42, .31, .17, and .25. Between the arithmetic reasoning and computation, the correlations were .72, .52, .33, and.64. The teachers' estimates of reading seemed to relate more closely with the reasoning part of the arithmetic standardized test than with the computation part. Important are the differences in correlations between the groups. The Dec-VM group, which had shown the lowest pretest scores of intelligence and arithmetic computation and reasoning showed a significant difference too on these key correlations. A difference may also be seen in the correlation between the place value subtest and intelligence and arithmetic tests.

Although certain differences among selected test score correlations are demonstrable among the four treatment groups, these may be more a result of previous group differences than of treatment effects.

TABLE 57

INTERCORRELATIONS OF SCORES OF GRADE FOUR STUDENTS

Non-Dec Treatment N=43[1] Non-Comp Treatment N=52[2]

Variable[3]	(9)	(11)	(12)	(13)	(14)	(15)	(16)	(18)	(19)	(24)	(16)	(28)
(9) Age		-17	-10	88	94	83	-20	-21	-22		-07	00
(11) Reading Estimate (Tr)	-16		55	-11	-06	-01	64	31	38		60	37
(12) Arith. Estimate (Tr)	-16	65		00	04	-09	45	45	53		44	55
(13) Visual Pretest	44	09	13		88	73	-00	-01	-04		-05	21
(14) Auditory Pretest	45	04	06	92		77	-03	-16	-19		-02	07
(15) Motor Coord. Pretest	39	27	25	81	82		-03	-16	-16		06	-06
											NON-COMP	
(16) CTMM Pretest	-10	75	56	17	04	22		25	42		49	35
	NON-DECIMAL											
(18) Stan-Arith Comp.Prtest	-04	42	36	18	15	19	51		52		26	59
(19) StanArith Reas. Prtest	13	43	24	22	22	27	52	72			34	
(24) Non-Lec.Posttest I	04	54	40	27	28	38	65	63	65			
(26) Geometry Posttest I												37
(28) Place Value Posttest I	-08	65	45	12	13	19	68	53		68		

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[1] With an N=43, r must equal .32 to be significant at the .01 level.

[2] With an N=52, r must equal .35 to be significant at the .01 level.

[3] Numbered as in Appendix E.

TABLE 58

INTERCORRELATIONS OF SCORES OF GRADE FOUR STUDENTS

Dec-VM Treatment N-42[1] Dec-Reg Treatment N=48[2]

Variable	(9)	(11)	(12)	(13)	(14)	(15)	(16)	(18)	(19)	(28)
(9) Age		-07	-10	07	15	11	-39	-12	00	-03
(11) Rdg. Estimate (Tr)	-13		45	-15	-02	-16	58	25	48	34
(12) Arith Estimate (Tr)	01	44		-04	-01	-01	40	46	52	26
(13) Visual Pretest	07	-03	-19		93	92	-05	-02	05	13
(14) Auditory Pretest	09	07	-22	78		90	-02	04	11	15
(15) Motor Coord. Pretest	01	22	03	65	70		-08	07	14	09
(16) CTMM Pretest	-32	56	52	-13	-14	11		46	DEC-REG 55	48
(18) Stan Arith Comp. Pretest	DEC-VM 10	17	39	07	18	00	49		64	32
(19) Stan Arith Reas. Pretest	11	51	25	02	03	19	57	33		
(28) Place Value Posttest I	-06	30	29	05	-06	08	35	44		

[1] With an N=42, r must equal .39 to be significant at the .01 level.

[2] With an N=48, r must equal .36 to be significant at the .01 level.

[3] Numbered as in Appendix E.

The degree of relationship between two variables which is represented by the correlation coefficient may be explained by the following analysis by Ferguson:

> In general, in attempting to conceptualize the degree of relationship represented by a correlation, it is more meaningful to think in terms of the square of the correlation coefficient instead of the correlation itself...Thus, a correlation of .10 represents a 1% association, a correlation of .50 represents a 25% association and the like... Whether a functional relationship can be regarded as a causal relationship is a matter of interpretation. (1959, p. 108)

The degrees of relationship depicted in Tables 56 through 58 are as follows:

Table	Lower Boundary Correlation Coefficients	Lower Boundary Percent of Relationship
56	.25	6.5
	.24	6.0
57	.32	9.9
	.35	12.5
58	.39	15.4
	.36	13.0

Though the percentage of relationship figures in the right-hand column appear low, it should be recalled that the correlation coefficients from which they are derived are significant to the .01 level.

The relevance of investigation of intercorrelations must be judged by each researcher. Speculations may also be made regarding their interpretation in education. No claims are made for substitution of causality for correlation. At best, the relationships herein depicted may lead to new conceptualizations and perhaps emphasize new directions for further study.

> Hypothesis 23. There are no significant differences among fourth grade students' scores on the Non-Decimal Test (Posttest I) and the Non-Decimal Test (Posttest II).

The Wilcoxon Matched-Pair Signed-Rank Test was used to analyze the group's performance on the two renditions of the Non-Decimal Test. This non-parametric test utilizes both direction and magnitude of score differences. Accordingly, a one-tailed test was used to compare the sums of the positive and negative ranks of the differences (Wilcoxon, Katti, Wilcox, 1963).

Table 59 shows the data and some of positive and negative ranks. The null hypothesis was H_0: the scores of fourth graders on the retention test were not significantly lower than scores on the first posttest. In terms of the Wilcoxon Test, the sum of the positive ranks equals the sum of the negative ranks.

Probability for the smaller of the like ranks to be less than 256 for $n=38$ is less than .0493. The number of pairs 38 is the original number 43 minus the number of pairs with a difference equal to zero.

The Wilcoxon Test leads to the rejection of the null hypothesis and the proposed acceptance of the alternative hypothesis. Fourth grade students' scores on the Non-Decimal retention test are significantly lower than scores on the first posttest.

TABLE 59

WILCOXON MATCHED PAIRS SIGNED RANKS TEST NON-DECIMAL TEST-POSTTEST I AND POSTTEST II GRADE FOUR (N=43)

Score Differences	Rank of Negative Differences	Rank of Positive Differences
+ 2		9.5
+24		38
- 8	31	
-11	33	
- 3	14	
- 7	28.5	
+ 1		3.5
- 7	28.5	
- 7	28.5	
- 3	14	
-14	36	
-11	33	
- 4	18.5	
- 6	24.5	
- 4	18.5	
+12		3.5
- 4	18.5	
- 2	9.5	
- 1	3.5	
+ 3		14
- 4	18.5	
+ 2		9.5
- 6	24.5	
- 2	9.5	
- 1	3.5	
- 2	9.5	
+ 2		9.5
+ 4		18.5
- 1	3.5	
- 4	18.5	
- 5	22	
+ 7		28.5
- 6	24.5	
-15	37	
+ 1		3.5
-11	33	
- 6	24.5	
	568.0	138.0

CHAPTER IV

FINDINGS FOR GRADE SIX

Pretest Data

During the pretest period, grade six students were given the California Test of Mental Maturity - Short Form, and the computation and reasoning subtests of the Stanford Arithmetic Achievement Test Level II. The means and standard deviations on these and other tests for the Non-Dec treatment are shown in Table 60 with and without the test constructor class. The representative nature of the sixth grade test constructor class is demonstrated by the similarity of mean scores in both cases.

Table 61 displays the means and standard deviations of the Non-Comp, Dec-VM, and Dec-Reg treatments for tests taken by those groups.

TABLE 60

MEANS AND STANDARD DEVIATIONS ON TEST INSTRUMENTS
GRADE SIX - TREATMENT NON-DEC

	Without Test Constructor Class N=47		With Test Constructor Class N=70	
	Mean	S.D.	Mean	S.D.
CTMM (Pretest)	114.57	13.52	112.80	12.53
CTMM (Posttest)	115.53	14.28	114.49	14.27
Stanford Test 1-X (Pretest)	20.40	5.30	19.04	5.65
Stanford Test 1-W (Posttest I)	19.94	6.92	19.49	6.73
Stanford Test 1-X (Posttest II)	22.74	6.80	21.13	7.35
Stanford Test 2-X (Pretest)	16.57	5.98	16.01	5.74

TABLE 60 -- Continued

	Without Test Constructor Class N=47 Mean	S.D.	With Test Constructor Class N=70 Mean	S.D.
Stanford Test 2-W (Posttest I)	17.65	6.45	16.59	6.30
Stanford Test 2-X (Posttest II)	19.85	6.83	18.41	7.22
Non-Decimal Test (Posttest I)	30.04	7.67	30.04	15.50
Non-Decimal Test (Posttest II)	29.40	8.94	29.41	9.10
Place Value Subtest (Pretest)	5.04	1.68	4.66	1.77
Place Value Subtest (Posttest I)	5.32	1.87	4.94	1.99
Place Value Subtest (Posttest II)	5.45	2.12	5.09	2.08

TABLE 61

MEANS AND STANDARD DEVIATIONS ON TEST INSTRUMENTS GRADE SIX - TREATMENTS NON-COMP, DEC-VM, DEC-REG

	Treatment Non-Comp N=53		Treatment Dec-Vm N=54		Treatment Dec-Reg N=45	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
CTMM (Pretest)	110.98	11.35	107.70	17.34	103.33	14.53
CTMM (Posttest)	116.94	10.03	108.26	17.11	105.78	14.37
Stanford Test 1-X (Pretest)	17.53	4.84	13.78	5.22	15.73	6.15
Stanford Test 1-W (Posttest I)	18.04	5.61	14.85	4.76	20.18	7.41
Stanford Test 1-X (Posttest II)	20.87	5.83	15.42	5.72	20.76	7.01
Stanford Test 2-X (Pretest)	16.81	5.08	12.89	4.78	13.20	4.76
Stanford Test 2-W (Posttest I)	17.58	5.84	14.57	5.75	14.04	5.98
Stanford Test 2-X (Posttest II)	17.94	5.56	14.61	5.47	15.18	6.11
Place Value Subtest (Pretest)	4.64	1.61	4.28	1.77	4.42	1.57
Place Value Subtest (Posttest I)	5.04	1.86	4.80	1.96	4.64	2.27
Place Value Subtest (Posttest II)	4.89	1.67	4.87	1.78	4.87	1.83
Geometry Unit Test	28.55	3.66				

Visual examination alone of the data of Tables 60 and 61 does not reveal whether the treatment groups may be considered statistically equal at the start of this experiment. In order to ascertain equality of treatment means, the analysis of variance must be employed for those pretests. The appropriateness of using this analysis of variance was tested by means of the Hartley Maximum-F Test of Homogeneity of Variance.

The F-ratios for the CTMM (Posttest) and the Stan-Test 1-W (Posttest I), tests to be considered later by means of analyses of covariance, are slightly above the ordinarily acceptable levels of significance. Winer (1962) does not regard slight departures from equality of population variances as troublesome to the researcher because of the robustness of the F-tests and the positive bias in the use of the larger N of unequal groups. For all analyses of variance and covariance of sixth grade treatment groups with respect to those tests listed in Table 62, the variances are considered to be homogeneous.

Pretest of Intelligence

The equality of group means of intelligence scores was tested by analysis of variance of the CTMM (Pretest). Table 63 shows the distribution of sums of squares for this analysis.

TABLE 62

HARTLEY MAXIMUM-F TEST OF HOMOGENEITY OF VARIANCE OF TEST SCORES OF SIXTH GRADE TREATMENTS

	Largest Variance	Smallest Variance	Numbers in Both Groups	$\frac{s^2\text{max}}{s^2\text{min}}$	F ** .05	F *** .01
CTMM (Pretest)	300.6	128.9	54.47	2.33	2.03	2.4
CTMM (Posttest)	292.8	100.6	54.53	2.91	2.03	2.4
Stanford Test 1-Form X (Pretest)	37.8	23.4	45.53	1.62	2.05	2.4
Stanford Test 1-Form W (Posttest I)	54.9	22.7	45.54	2.42	2.03	2.4
Stanford Test 1-Form X (Posttest II)	49.1	32.7	45.54	1.50	2.03	2.4
Stanford Test 2-Form X (Pretest)	35.7	22.7	47.45	1.57	2.14	2.6

TABLE 62 -- Continued

	Largest Variance	Smallest Variance	Numbers in Both Groups	$\frac{s^2max}{s^2min}$	F ** .05	F *** .01
Stanford Test 2-Form W (Posttest I)	41.6	33.1	47.54	1.26	2.03	2.4
Stanford Test 2-Form X (Posttest II)	46.6	29.9	47.54	1.56	2.03	2.4
Place Value Subtest (Posttest I)	3.50	2.79	47.53	1.25	2.05	2.4
Place Value Subtest	4.49	2.79	47.53	1.61	2.05	2.4

** .05
*** .01

TABLE 63

ANALYSIS OF VARIANCE - CTMM (Pretest)
Grade 6 (N=199)

Source	ss	df	Mean Square	F Ratio
Among	3195	3	1065	5.1493**
Within	40336	195	206	
Total	43531	198		

The F-ratio in Table 63, significant to the .05 level, indicates that the treatment means are indeed unequal. The Scheffe Test for comparisons among treatment means was therefore employed to determine how the various treatment groups related on the intelligence score criterion. Table 64 shows the data used in the Scheffe comparisons (Appendix I).

TABLE 64

SCHEFFE COMPARISONS BETWEEN MEANS ON CTMM (Pretest)

	Treatment Means						S.E.	
Comparison	114.57	110.98	107.70	103.33	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	3.59	3.03	1.185
(1)vs(3)	1	0	-1	0	2	6.87		2.267
(1)vs(4)	1	0	0	-1	2	11.24		3.709***
(2)vs(3)	0	1	-1	0	2	3.28		1.082
(2)vs(4)	0	1	0	-1	2	7.65		2.525*
(3)vs(4)	0	0	1	-1	2	4.37		1.442
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	7.34	7.42	.989
(1)vs(3) +(4)	2	0	-1	-1	6	18.11	5.25	3.450***

Table 64 reveals the ranking of treatment groups by means of intelligence test scores attained on the CTMM (Pretest) for grade six to be as follows:

Group Number	Treatment Ran	Rank
(1)	Non-Dec	1
(2)	Non-Comp	2
(3)	Dec-VM	3.5
(4)	Dec-Reg	3.5

The four treatment groups began the experiment unequal on this measure of intelligence. There was the need therefore to equate these groups statistically. The use of analysis of covariance with the CTMM as covariant was used where appropriate.

Pretest of Arithmetic Computation

Table 65 presents the sums of squares distribution for the analysis of variance of Stan-Test 1-Form X, used as the pretest of arithmetic computation.

TABLE 65

ANALYSIS OF VARIANCE - STAN-TEST 1, FORM X
(Pretest)-COMPUTATION GRADE SIX (N=199)

Source	SS	df	Mean Square	F Ratio
Among	1183	3	394.350	13.681***
Within	5621	195	28.824	
Total	6804	198		

The significant F-ratio in Table 65 suggested the possibility of making comparisons among the means by the Scheffe Test. Table 66 displays the data for these comparisons.

TABLE 66

SCHEFFE COMPARISONS BETWEEN MEANS OF STAN-TEST 1, FORM X (Pretest)

	Treatment Means						S.E.	
Comparison	20.40	17.53	13.78	15.73	Σa_i^2	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	2.87	1.13	2.540*
(1)vs(3)	1	0	-1	0	2	6.62		5.859***
(1)vs(4)	1	0	0	-1	2	4.67		4.133***
(2)vs(3)	0	1	-1	0	2	3.75		3.319**
(2)vs(4)	0	1	0	-1	2	1.80		1.593
(3)vs(4)	0	0	1	-1	2	-1.95		-1.726
(2)vs(1)+(3)+(4)	-1	3	-1	-1	12	2.68	2.77	.968
(1)vs(3)+(4)	2	0	-1	-1	6	11.24	1.96	5.735***

The Scheffe Test for treatment means on the pretest of arithmetic computation for grade six (Table 66) indicated that the students in treatment Non-Dec began with a markedly superior ability in arithmetic computation over all the other groups. Here then is further indication of need to employ analysis of covariance for posttest computation scores.

Pretest of Arithmetic Reasoning

Comparison of means of grade six treatment groups for the arithmetic reasoning test, STAN-Test 2, Form X (Pretest) is shown in Table 67.

TABLE 67

ANALYSIS OF VARIANCE STAN-TEST 2 X (Pretest)
ARITHMETIC REASONING - GRADE SIX
(N=199)

Source	SS	df	Mean Square	F-Ratio
Among	674	3	224.575	8.438**
Within	5190	195	26.616	
Total	5864	198		

Significant differences among treatment means indicated the appropriateness of the Scheffe comparison of means. Data are shown in Table 68 for this statistical test.

TABLE 68

SCHEFFE COMPARISONS BETWEEN MEANS ON STAN-TEST 2-FORM X (Pretest)

	Treatment Means						S.E.	
Comparison	16.57	16.81	12.89	13.20	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	-.24	1.09	-.220
(1)vs(3)	1	0	-1	0	2	3.68		3.376**
(1)vs(4)	1	0	0	-1	2	3.37		3.092**
(2)vs(3)	0	1	-1	0	2	3.92		3.596***
(2)vs(4)	0	1	0	-1	2	3.61		3.312**
(3)vs(4)	0	0	1	-1	2	-.31		-.284
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	7.77	2.67	2.910**
(1)vs(3) +(4)	2	0	-1	-1	6	7.05	1.89	3.730***

Table 68 reveals that groups Non-Dec and Non-Comp began the experiment with scores superior to those of students in treatments Dec-VM and Dec-Reg.

Summary of Analysis of Pretest Data

On all three pretests, the four treatment groups could not be considered to have equal means. Of particular importance to this experiment, the Non-Dec treatment group means were significantly superior to most other groups on all three measures. To assess the effects of any treatment on the criteria of arithmetic computation and reasoning, the groups must be equated statistically by means of analysis of covariance.

Hypotheses Concerning Grade Six

The even-numbered hypotheses will be considered one at a time in this chapter. Supporting data will be presented for rejection or non-rejection of each hypothesis.

Hypothesis 2: There are no significant differences for scores on STAN-Test 1, Form W (Posttest I) - Computation - among groups of sixth grade students receiving the four treatments.

Analysis of Stan-Test 1-W-Computation-(Posttest I) was done using as covariants the CTMM (Pretest) and the Stan-Test 1-X (Pretest). Data derived from the original scores for use in this analysis is shown in Table 69.

The highly significant F-ratio in Table 69 obtained despite statistical correction by means of two covariants, led to rejection of the hypothesis of equal treatment means on scores of this computation test.

TABLE 69

ANALYSIS OF COVARIANCE STAN-TEST 2-W (Posttest I) COMPUTATION - COVARIANTS: CTMM (Pretest) and STAN-TEST 1-X (Pretest) GRADE 6 (N=199)

Source	SS	SS Due to Regression	SS About Regression	df	Mean Square	Ratio
Among	923					
Within	7454	4159	3295	193	17.075	12.224***
Total	8377	4455	3921	196		
Diff. for Testing Among Adjusted Treatment Means			626	3	208.723	

The coefficients, their standard errors, and computed t-values as shown for each covariant in Table 70.

TABLE 70

COEFFICIENTS FOR COVARIANTS FOR DATA OF TABLE 69

	CTMM (Pretest)			Stan-Test 1-X (Pretest)		
Source	Coeff.	S.E.	t	Coeff.	S.E.	t
Within	.0912	.0241	3.7836***	.7071	.0646	10.9486***
Total	.0631	.0255	2.4724**	.7112	.0646	11.0085***

The significant t-values in Table 70 indicated that the within-treatment and total regression for each covariant was non-zero.

Adjusted treatment means, their standard errors, and adjusted variances are shown in Table 71.

TABLE 71

ADJUSTED TREATMENT MEANS, STANDARD ERRORS AND ADJUSTED VARIANCES FOR DATA OF TABLE 69

Treatment	N	Mean	Adj. Mean	S.E. Adj. Mean	Adj. Var.	Adj. Var. max / Adj. Var. min
Non-Dec	47	19.96	16.89	.6349	18.95	1.10
Non-Comp	53	18.04	17.35	.5693	17.18	
Dec-VM	54	14.85	17.12	.5898	18.78	
Dec-Reg	45	20.18	21.46	.6278	17.36	

Homogeneity of adjusted variances is indicated by the non-significant F-ratio in the application of the Hartley Max-F test in Table 71.

Linearity of overall regression in the analysis of covariance data for grade six was assumed in cases of more than one covariant. As explained in Chapter IV, derived data did not permit separation of sums of squares into components needed to test this assumption statistically.

The Scheffe Test was used to analyze and determine which treatment means were not equal. The test enabled many comparisons to be made besides comparisons of all possible pairs. Again in the following analyses, Treatment Non-Comp was compared to the average of the other three computational treatments and Treatment Non-Dec was compared to the average of Dec-VM and Dec-Reg in addition to the usual comparison by pairs.

TABLE 72

SCHEFFE COMPARISONS BETWEEN ADJUSTED MEANS STAN-TEST 1, FORM W, (POSTTEST I)

	Treatment Means						S.E.	
Comparison	16.89	17.35	17.12	21.46	$\sum a_i^2$	d_i	of d_1	t
(1)vs(2)	1	-1	0	0	2	-.46	.87	-.528
(1)vs(3)	1	0	-1	0	2	-.23		-.264
(1)vs(4)	1	0	1	-1	2	-4.57		-5.264***
(2)vs(3)	0	1	-1	0	2	.23		.264
(2)vs(4)	0	1	0	-1	2	-4.11		4.718***
(3)vs(4)	0	0	1	-1	2	-3.88		-4.454***
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	-3.42	2.133	-1.603
(1)vs(3) +(4)	2	0	-1	-1	6	-4.80	1.59	-3.181**

Table 72 shows the data used in this application of the Scheffe Test.

The treatments may be ranked as follows:

Group Number	Treatment	Rank
(4)	Dec-Reg	1
(1)	Non-Dec	3
(2)	Non-Comp	3
(3)	Dec-VM	3

Treatment Dec-Reg showed a marked superiority over the other three treatment groups on this posttest of arithmetic computation. In fact, the treatment Non-Dec mean score was clearly inferior to the mean scores of the groups studying decimal numeration, as can be noted in the data shown on the last line of Table 72.

Hypothesis 4: There are no significant differences for scores on STAN-Test 2, Form W (Posttest I) - Arithmetic Reasoning - among groups of sixth grade students receiving the four treatments.

Derived data for analysis of covariance for the test of arithmetic reasoning, STAN-Test 2, Form W (Posttest I) for grade six is shown in Table 73.

TABLE 73

ANALYSIS OF COVARIANCE STAN-TEST 1, FORM W (Posttest I) - ARITHMETIC REASONING - COVARIANTS: CTMM (Pretest) and STAN-TEST 2, FORM X (Pretest) GRADE SIX (N=199)

Source	SS	SS Due to Regression	SS About Regression	df	Mean Square	F Ratio
Among	544					
Within	7016	4473	2544	193	13.180	.122
Total	7561	5012	2549	196		
Diff. for Testing Among Adjusted Treatment Means			5	3	1.608	

The non-significant F-ratio of Table 73 supported the hypothesis of equal means.

The non-zero within-treatment and total regression coefficients for each covariant are exhibited by highly significant t values in Table 74.

TABLE 74

COEFFICIENTS FOR COVARIANTS FOR DATA IN TABLE 73

	CTMM	(Pretest)		STAN-Test 2, Form X		(Pretest)
Source	Coeff.	S.E.	t	Coeff.	S.E.	t
Within	.0801	.0229	3.5056***	.7746	.0637	12.1530***
Total	.0818	.0223	3.6748***	.7676	.0607	12.6534***

Homogeneity of adjusted variances was substantiated by the non-significant F-ratio shown in Table 75.

TABLE 75

ADJUSTED TREATMENT MEANS AND STANDARD ERRORS STAN-Test 2, FORM W (POSTTEST I) GRADE 6

Treatment	N	Mean	Adj. Mean	S.E. Adj. Mean	Adj. Var.	Adj. Var. max. / Adj. Var. min.
Non-Dec	47	17.66	15.91	.5394	13.67	1.10
Non-Comp	53	17.58	15.94	.5094	13.75	
Dec-VM	54	14.57	16.23	.5059	13.82	
Dec-Reg	45	14.04	15.81	.5521	13.72	

Analysis of covariance revealed that the four treatment groups had statistically equal group means on the test of arithmetic reasoning which was given immediately after the teaching period. This analysis was affirmed as appropriate by the testing of assumptions underlying the analysis of variance.

Hypothesis 6: There are no significant differences for scores on STAN-Test 1, Form X (Posttest II) - Computation - among groups of sixth grade students receiving the four treatments.

The CTMM (Posttest) was administered near in time to the retention tests, whereas the CTMM (Pretest) had been administered about twelve weeks earlier. There was a question as to the appropriateness of using one or the other as covariant in the analysis of the retention test of computation. Parallel analyses were tried, using each as one of the three covariants. Tables 76 and 77 show the derived data for analysis in both cases.

TABLE 76

ANALYSIS OF COVARIANCE: STAN-TEST 1, FORM X (Posttest II) - COMPUTATION - COVARIANT: CTMM (Pretest), STAN-TEST 1, FORM X (Pretest) and STAN-TEST 1, Form W (Posttest I) GRADE SIX (N=199)

Source		SS Due to Regression	SS About Regression	df	Mean Square	F Ratio
Among	1542					
Within	7792	4820	2971	192	15.482	3.974***
Total	9335	6178	3157	195		
Diff. for Testing Among Adjusted Treatment Means			185	3	61.527	

The F-ratios were significant in the analysis of covariance for the retention test of arithmetic computation when adjustments of the treatment means were made using the CTMM (Pretest as in Table 76 or the CTMM (Posttest) as in Table 77, leading to the rejection of the null hypothesis: H_o: All μ_1 are equal.

TABLE 77

ANALYSIS OF COVARIANCE: STAN-TEST 1, FORM X (Posttest II) - COMPUTATION - COVARIANT: CTMM (Posttest), STAN-TEST 1, FORM X (Pretest and STAN-TEST 1, FORM W (Posttest I) - GRADE SIX (N=199)

Source	SS	SS Due to Regression	SS About Regression	df	Mean Square	F Ratio
Among	1542					
Within	7792	4213	3579	192	18.642	7.962***
Total	9935	5310	4024	195		
Diff. for Testing Among Adjusted Treatment Means			445	3	148.426	

A possible argument for use of the CTMM (Posttest) rather than CTMM (Pretest) in covariance analysis was obtained from Tables 78 and 79.

Total regression coefficients for the CTMM (Pretest) in Table 78 has a non-significant value, indicating the test contributed little to analysis of the retention test of arithmetic computation.

The data of Table 97 shows that the covariant, STAN-Test 1-W, Posttest I, contributes no significant value to this regression analysis. This might mean that the intelligence test, CTMM, and the retention test of computation are testing the same factors.

TABLE 78

COEFFICIENTS FOR COVARIANTS FOR DATA IN TABLE 76

Source	CTMM (Pretest)			STAN-Test 1, X (Pretest)			STAN-Test 1, W (Posttest I)		
	Coeff.	S.E.	t	Coeff.	S.E.	t	Coeff.	S.E.	t
Within	.0486	.0238	2.0407**	.3500	.0783	4.4698***	.4840	.0685	7.0616***
Total	.0362	.0233	1.5504	.3952	.0739	5.3457***	.5156	.0643	8.0244***

TABLE 79

COEFFICIENTS FOR COVARIANTS FOR DATA IN TABLE 77

Source	CTMM (Posttest)			STAN-Test 1, X (Pretest)			STAN-Test 1, W (Posttest I)		
	Coeff.	S.E.	t	Coeff.	S.E.		Coeff.	S.E.	
Within	.0913	.0298	3.0643***	.5941	.0766	7.7581***	.1530	.0873	1.7532
Total	.0703	.0308	2.2804**	.6609	.0754	8.7686***	.1503	.0910	1.6522

Effects on treatment means of both sets of covariants appear in Tables 80 and 81.

TABLE 80

ADJUSTED TREATMENT MEANS, STANDARD ERRORS AND ADJUSTED VARIANCES - STAN-TEST 1, FORM X (Posttest II) - GRADE SIX

Treatment	N	Mean	Adj. Mean	S.E. Adj. Mean	Adj. Var.	Adj. Var. max. / Adj. Var. min.
Non-Dec	47	22.74	20.33	.6104	17.51	1.17
Non-Comp	53	20.87	20.55	.5446	15.72	
Dec-VM	54	15.42	18.13	.5657	17.28	
Dec-Reg	45	20.76	20.41	.6404	18.46	

TABLE 81

ADJUSTED TREATMENT MEANS, STANDARD ERRORS AND ADJUSTED VARIANCES - STAN-TEST 1, FORM X (Posttest II) - GRADE SIX

Treatment	N	Mean	Adj. Mean	S.E. Adj. Mean	Adj. Var.	Adj. Var. max. / Adj. Var. min.
Non-Dec	47	22.74	19.99	.6650	20.78	1.068
Non-Comp	53	20.87	19.65	.6087	19.64	
Dec-VM	54	15.43	17.83	.6137	20.34	
Dec-Reg	45	20.76	22.18	.6574	19.45	

Homogeneity of adjusted variances was upheld by the non-significant F-ratios as shown in the right-hand columns of Tables 80 and 81.

Tables 82 and 83 list data for the Scheffe Tests with both sets of data to detect unequal means.

TABLE 82

SCHEFFE COMPARISONS BETWEEN ADJUSTED MEANS FOR DATA OF TABLE 78

	Adj. Treatment Means						S.E.	
Comparison	20.33	20.55	18.13	20.41	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	-.22	.829	-.265
(1)vs(3)	1	0	-1	0	2	2.20		2.653*
(1)vs(4)	1	0	0	-1	2	-.08		.096
(2)vs(4)	0	1	0	-1	2	.14		.169
(3)vs(4)	0	0	1	-1	2	-2.28		-2.750*
(2)vs(1)+(3)+(4)	-1	3	-1	-1	12	2.78	2.030	1.369
(1)vs(3)+(4)	2	0	-1	-1	6	2.12	1.436	1.476

TABLE 83

SCHEFFE COMPARISONS BETWEEN ADJUSTED MEANS FOR DATA OF TABLE 79

	Adj. Treatment Means						S.E.	
Comparison	19.99	19.65	17.83	22.18	Σa_i^2	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	.34	.934	.364
(1)vs(3)	1	0	-1	0	2	2.16		2.313#
(1)vs(4)	1	0	0	-1	2	-2.19		-2.345#
(2)vs(3)	0	1	-1	0	2	1.82		1.949
(2)vs(4)	0	1	0	-1	2	-2.53		-2.710*
(3)vs(4)	0	0	1	-1	2	-4.35		-4.659***
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	1.05	2.29	.459
(1)vs(3) +(4)	2	0	-1	-1	6	.00	1.62	.002

Tables 82 and 83 disclosed the possible ranking of the four groups to be as follows:

Group No.	Treatment	Table 82	Table 83
(4)	Dec-Reg	2	1
(2)	Non-Comp	2	2.5
(1)	Non-Dec	2	2.5
(3)	Dec-VM	4	4

The differences in the two rankings shown above are relatively small. One clear conclusion which may be noted is the inferiority of the adjusted treatment mean of the grade six Dec-VM treatment group.

Hypothesis 8: There are no significant differences for scores on STAN-Test 2 Form X (Posttest II) - Arithmetic Reasoning - among groups of sixth grade students receiving the four treatments.

Two parallel analyses were made for the testing of this hypothesis in the manner of Hypothesis 6. The retention test of arithmetic reasoning - STAN-Test 2, Form X (Posttest II) was the dependent variable in the following analyses of covariance. First, the CTMM (Pretest), STAN-Test 2, Form X (Pretest), and STAN-Test 2, Form W (Posttest I) were the covariants (Table 84). For the second analysis, the CTMM (Posttest), STAN-Test 2, Form X (Pretest) and STAN-Test 2, Form W (Posttest I) were the covariants (Table 85).

TABLE 84

ANALYSIS OF COVARIANCE: STAN-TEST 2-X (Posttest II)
ARITHMETIC REASONING - COVARIANT: CTMM (Pretest),
STAN-TEST 2-X (Pretest) and STAN-TEST 2-W (Posttest I)
GRADE SIX (N=199)

Source	SS	SS Due to Regression	SS About Regression	df	Mean Square	F Ratio
Among	883					
Within	6988	5343	1645	192	8.569	4.027***
Total	7871	6123	1749	195		
Diff. for Testing Among Adjusted Treatment Means			104	3	34.509	

TABLE 85

ANALYSIS OF COVARIANCE: STAR-TEST 2-X (Posttest II)
COVARIATES: CTMM (Posttest), STAR-TEST 2-X (Pretest)
STAR-TEST 2-W (Posttest I) - GRADE SIX (N=199)

Source	SS	SS Due to Regression	SS About Regression	df	Mean Square	F Ratio
Among	883					
Within	6988	5338	1650	192	8.595	4.953***
Total	7871	6093	1778	195		
Diff. for Testing Among Adjusted Treatment Means			128	3	42.573	

On the basis of significant F-ratios appearing for the derived data in Tables 84 and 85, the null hypothesis of equal means, $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$, was rejected in favor of the alternate hypothesis, H_1: Some μ_i are not equal.

Raw-score within-treatment and total regression coefficients are shown in Tables 86 and 87.

TABLE 86

COEFFICIENTS FOR COVARIANTS FOR DATA IN TABLE 84

Source	CTMM (Pretest) Coeff.	S.E.	t	STAN-Test 2,X (Pretest) Coeff.	S.E.	t	STAN-Test 2, W (Posttest I) Coeff.	S.E.	t
Within	.0681	.0191	3.5828***	.5386	.0683	7.8887***	.3390	.0580	5.814***
Total	.0717	.0191	3.7505***	.5489	.0679	8.0833***	.3341	.0593	5.6326***

TABLE 87

COEFFICIENTS FOR COVARIANTS FOR DATA IN TABLE 85

Source	CTMM (Posttest) Coeff.	S.E.	t	STAN-Test 2,X (Pretest) Coeff.	S.E.	t	STAN-Test 2, W (Posttest I) Coeff.	S.E.	t
Within	.0700	.0200	3.4932***	.5217	.0701	7.4427***	.3471	.0577	6.0190***
Total	.0665	.0200	3.2589***	.5339	.0709	7.5301***	.3491	.0592	5.8903***

Homogeneity of adjusted variances is demonstrated in the non-significant F-ratios for the Hartley Max-F test, shown in the right hand columns of Tables 88 and 89.

TABLE 88

ADJUSTED TREATMENT MEANS, STANDARD ERRORS, AND ADJUSTED VARIANCES FOR DATA OF TABLE 84

Treatment	N	Mean	Adj. Mean	S.E. Adj. Mean	Adj. Var.	$\frac{\text{Adj. Var. max.}}{\text{Adj. Var. min.}}$
Non-Dec	47	19.85	18.00	.4349	8.89	1.012
Non-Comp	53	17.94	16.24	.4107	8.94	
Dec-VM	54	14.61	16.26	.4082	9.00	
Dec-Reg	45	15.18	17.14	.4452	8.92	

TABLE 89

ADJUSTED TREATMENT MEANS, STANDARD ERRORS, AND ADJUSTED VARIANCES FOR DATA OF TABLE 85

Treatment	N	Mean	Adj. Mean	S.E. Adj. Mean	Adj. Var.	$\frac{\text{Adj. Var. max.}}{\text{Adj. Var. min.}}$
Non-Dec	47	19.85	18.12	.4335	8.83	1.019
Non-Comp	53	17.94	16.01	.4121	9.00	
Dec-VM	54	14.61	16.38	.4077	8.98	
Dec-Reg	45	15.18	17.14	.4462	8.96	

Data for the Scheffe Test of comparison of means was shown in Tables 90 and 91.

TABLE 90

SCHEFFE COMPARISONS BETWEEN ADJUSTED MEANS FOR DATA OF TABLE 84

	Adj. Treatment Means						S.E.	
Comparison	18.00	16.24	16.26	17.14	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	1.76	.617	2.852**
(1)vs(3)	1	0	-1	0	2	1.74		2.820**
(1)vs(4)	1	0	0	-1	2	.86		1.394
(2)vs(3)	0	1	-1	0	2	-.02		-.324
(2)vs(4)	0	1	0	-1	2	-.90		-1.459
(3)vs(4)	0	0	1	-1	2	-.88		-1.426
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	-2.72	1.511	-1.800
(1)vs(3) +(4)	2	0	-1	-1	6	2.60	1.069	2.432#

TABLE 91

SCHEFFE COMPARISONS BETWEEN ADJUSTED MEANS FOR DATA OF TABLE 85

	Adj. Treatment Means						S.E.	
Comparison	18.12	16.01	16.38	17.14	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	2.11	.618	3.414***
(1)vs(3)	1	0	-1	0	2	1.74		2.815**
(1)vs(4)	1	0	0	-1	2	.98		1.586
(2)vs(3)	0	1	-1	0	2	-.37		-.599
(2)vs(4)	0	1	0	-1	2	-1.13		-1.828
(3)vs(4)	0	0	1	-1	2	-.76		-1.230
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	-3.61	1.513	-2.386#
(1)vs(3) +(4)	2	0	-1	-1	6	2.72	1.070	2.542*

For the data of Tables 90 and 91 the approximate ranking for the treatment groups is as follows for the retention test of arithmetic reasoning:

Group Number	Treatment	Rank
(1)	Non-Dec	1
(4)	Dec-Reg	2
(2)	Non-Comp	3.5
(3)	Dec-VM	3.5

In addition, the following conclusions concerning treatment means may be made:

1) The weighted mean of the computation groups exceeds slightly that of the non-computation group on this test.

2) The mean of the non-decimal group exceeds that of the weighted mean score of the decimal groups.

Hypothesis 10: There are no significant differences for CTMM (Posttest II) scores when groups have been matched according to CTMM (Pretest) among sixth grade students.

The analysis of the CTMM (Posttest) using the CTMM (Pretest) as covariant was considered to be significant in this experiment for the following reasons:

1) Similar factors may be measured by the intelligence tests (CTMM) and by the arithmetic achievement tests (Stanford Arithmetic Scores).

2) The experiment itself may have influenced the scores on the intelligence test (CTMM).

3) The intelligence test scores may provide an additional measure of change resulting from the experimental treatments.

Table 92 shows the expanded sums of squares. Derived data shown therein supplied the information for testing of the assumptions underlying the covariance test. The partial adjusted sums of squares, S_1, listed with their degrees of freedom in Table 93 and the tests of assumptions may be seen in Table 94.

TABLE 92

ANALYSIS OF CTMM (Posttest) - COVARIANT: CTMM (Pretest) GRADE SIX (N=199)

Source	$\sum x^2$	$\sum xy$	$\sum y^2$	df	$y^2 - \frac{(\Sigma xy)^2}{\Sigma x^2}$	MS	F	Adj df	MS^1	F^1
Within										
Non-Dec.	8415	7702	9382		2333					
Non-Comp.	6703	4634	5227		2032					
Dec-VM	15927	13621	15514		3865					
Dec-Reg.	9290	8632	9086		1065					
					$S_1 = 9295$					
Between Groups	3195	3304	4365	3	$S_3 = 948$	1455	7.236***			
Within Groups	40336	34589	39209	195	$S_p = S_1 + S_2 =$ 9548			194	49.21	7.044***
Total	43531	37893	43574	198	S_1 10588	209		197		
Diff. for Testing Among Adjusted Treatment Means					$S_3 + S_4$ 1040			3	346.65	

$S_2 = S_p - S_1$; $S_4 = S_T - S_p - S_3$

TABLE 93

PARTITIONS OF VARIANCE FOR TESTS ON ASSUMPTIONS UNDERLYING ANALYSIS OF COVARIANCE OF CTMM (Posttest)

S_i	Value	df	df Interpretation	
S_1	9295	191	$\Sigma(n_i-1)-r-(k-1)$	r-number of regression coefficients
S_2	252	3	$k-1$	k-number of treatments
S_3	948	2	$k-2$	n-number of scores in one treatment
S_4	92	1	1	
S_T	10587	197	$\Sigma(n_i-1)-r-k-1$	

Test 2(b) of Table 94 disclosed that one of the assumptions was actually not upheld by this data. The assumption violated was that the between-treatment regression was linear. According to Winer (1962):

> If the within-class regression is linear, and if the covariate is not affected by the treatment, it is reasonable to expect that the between-class regression will be linear. (p.587)

Winer further stated that if regression is not linear, the interpretation of adjusted treatment means is difficult. However, in reference to this same matter, Dixon and Massey (1957) see no need to employ tests 2(a), 2(b), or 2(c) if test 2 is non-significant. For purposes of this analysis, the main requirements were considered upheld.

TABLE 94

TESTS ON ASSUMPTIONS UNDERLYING ANALYSIS OF COVARIANCE

Description	Test	F-Ratio
(1) Difference in Means	$F = \dfrac{\dfrac{S_3 + S_4}{k-1}}{\dfrac{S_1 + S_2}{\Sigma(n_i-1)-r}}$	7.044***
(2) Can one regression line be used for all observations, i.e., is the overall regression linear: If significant, use (a), then (b) and then (c)	$F = \dfrac{\dfrac{S_2 + S_3 + S_4}{2(k-1)}}{\dfrac{S_1}{\Sigma(n_i-1)-r-(k-1)}}$	.023
(a) Are the slopes of regression lines within treatment groups the same?	$F = \dfrac{\dfrac{S_2}{k-1}}{\dfrac{S_1}{\Sigma(n_i-1)-r-(k-1)}}$	1.726
(b) Is the between treatment regression linear?	$F = \dfrac{\dfrac{S_3}{k-2}}{\dfrac{S_1 + S_2}{\Sigma(n_i-1)-r}}$	9.632***
(c) If slopes are the same and regression for means is linear, are between treatment regression coefficients the same as within treatment regression coefficients?	$F = \dfrac{\dfrac{S_4}{1}}{\dfrac{S_1 + S_2}{\Sigma(n_i-1)-r}}$	1.870

The within-treatment regression coefficient, its standard error of estimate, and its t-value are .8575, .0349, and 24.5499*** respectively. The total regression coefficient, its standard error of estimate, and its t-value are .8705, .0351, and 24.7742***, respectively. Clearly, the t-values show the regression coefficient to be non-zero.

Table 95 shows adjusted treatment means, standard errors, and adjusted variances for data of Table 92.

TABLE 95

ADJUSTED TREATMENT MEANS, STANDARD ERRORS, AND ADJUSTED VARIANCES FOR DATA OF TABLE 92

Treatment	N	Mean	Adj. Mean	S.E. Adj. Mean	Adj. Var.	$\frac{\text{Adj. Var. max.}}{\text{Adj. Var. min.}}$
Non-Dec	47	115.53	110.93	1.04	50.84	1.035
Non-Comp	53	116.94	115.43	.97	49.87	
Dec-VM	54	108.26	109.55	.96	49.77	
Dec-Reg	45	105.78	110.82	1.07	51.52	

The F-ratio of Table 95 indicates homogeneity of adjusted variances.

Table 96 exhibits data for the Scheffe Test for comparisons of adjusted treatment means.

TABLE 96

SCHEFFE COMPARISONS BETWEEN ADJUSTED MEANS FOR DATA OF TABLE 92

	Adj. Treatment Means						S.E.	
Comparison	110.93	115.43	109.55	110.82	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	-4.50	1.48	-3.041**
(1)vs(3)	1	0	-1	0	2	1.38		.932
(1)vs(4)	1	0	0	-1	2	.11		.074
(2)vs(3)	0	1	-1	0	2	5.88		3.973***
(2)vs(4)	0	1	0	-1	2	4.61		3.115**
(3)vs(4)	0	0	1	-1	2	-1.27		-.858
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	14.99	3.625	4.135***
(1)vs(3) +(4)	2	0	-1	-1	6	1.49	2.563	.581

The comparison by pairs of means and the comparison of the non-computation group with the computation group's weighted mean indicated a highly significant degree of superiority of the Non-Comp group over all the others. The relevance of this finding and possible meaning in terms of this experiment are subject to the limitations imposed by the standard error of the test and its relevance for group mean analysis.

Hypothesis 12: There are no differences for difference scores between the Pretest and Posttest I STAN-Test 1 scores among groups of sixth grade students receiving the four treatments.

Because the variances for the scores of the four treatment groups proved to be homogeneous (see Appendix H) the data could be analyzed by means of the analysis of variance statistical test.

Table 97 shows the data for the analysis of variance of the difference scores on the STAN-Test 1 - Arithmetic Computation between the first rendition, the Pretest, and the second rendition (Posttest I).

TABLE 97

ANALYSIS OF VARIANCE - ARITHMETIC COMPUTATION DIFFERENCE SCORES - STAN-TEST 1 (Posttest I - Pretest) - GRADE SIX (N=199)

Source	SS	df	Mean Square	F-Ratio
Among	628	3	209.22	11.045***
Within	3693	195	18.94	
Total	4321			

Scheffe comparisons are shown in Table 98.

TABLE 98

SCHEFFE COMPARISONS BETWEEN MEANS FOR DATA OF TABLE 97

	Treatment Means						S.E.	
Comparison	-.468	.509	1.074	4.444	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	-.977	.870	-1.123
(1)vs(3)	1	0	-1	0	2	-1.542	.868	-1.776
(1)vs(4)	1	0	0	-1	2	-4.912	.907	-5.416***
(2)vs(3)	0	1	-1	0	2	-.565	.844	-.669
(2)vs(4)	0	1	0	-1	2	-3.935	.881	-4.467***
(3)vs(4)	0	0	1	-1	2	-3.370	.879	-3.834***
(2)vs(1)+(3)+(4)	-1	3	-1	-1	12	-3.523	2.11	-1.670
(1)vs(3)+(4)	2	0	-1	-1	6	-6.454	1.53	-4.218***

An examination of Table 98 indicates that the mean of the difference scores of treatment Dec-Reg far exceeded those of the other three groups. Furthermore, the difference scores of the other three treatment groups are statistically equal.

Hypothesis 14. There are no differences for difference scores between the Pretest and Posttest I STAN-Test 2 among groups of sixth grade students receiving the four treatments.

Data for the analysis of variance of the difference scores on the arithmetic reasoning test, STAN-Test 2, between the Pretest and the Posttest I are shown in Table 99.

TABLE 99

ANALYSIS OF VARIANCE - ARITHMETIC REASONING DIFFERENCE SCORES - STAN-TEST 2 (Posttest I - Pretest) - GRADE SIX (N=199)

Source	SS	df	Mean Square	F-Ratio
Among	27	3	9.022	.641
Within	2746	195	14.085	
Total	2773	198		

The non-significant F-ratio of Table 99 indicated that the null hypothesis of equal treatment means was not rejected.

Hypothesis 16: There are no significant differences for difference scores between the Posttest I and Posttest II STAN-Test 1 scores among the sixth grade students receiving the four treatments.

The sums of squares data is exhibited in Table 100 for the analysis of variance for the arithmetic computation test of score differences between the second rendition, Posttest I, and the third rendition, Posttest II, of the STAN-Test 1.

TABLE 100

ANALYSIS OF VARIANCE - ARITHMETIC COMPUTATION DIFFERENCE SCORES - STAN-TEST 1 (Posttest II - Posttest I) - GRADE SIX (N=199)

Source	SS	df	Mean Square	F-Ratio
Among	250	3	83.52	4.230***
Within	3851	195	19.74	
Total	4101	198		

The significant F-ratio of Table 100 indicates rejection of the null hypothesis of equal treatment means and therefore Scheffe comparisons of means were made. The data for these comparisons is shown in Table 101.

TABLE 101

SCHEFFE COMPARISONS BETWEEN MEANS FOR DATA OF TABLE 100

	Treatment Means						S.E.	
Comparison	2.8085	2.8302	.5741	.5778	$\sum a_i^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	-.0217	.937	-.023
(1)vs(3)	1	-1	0	0	2	2.2344		2.384#
(1)vs(4)	1	0	0	-1	2	2.2307		2.380#
(2)vs(3)	0	1	-1	0	2	2.2561		2.402#
(2)vs(4)	0	1	0	-1	2	2.2524		2.403#
(3)vs(4)	0	0	1	-1	2	-.0037		-.004
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	4.5302	2.295	1.974
(1)vs(3) +(4)	2	0	-1	-1	6	4.4651	1.623	2.751*

The Scheffe comparisons of Table 101 indicate an approximate ranking as follows:

Group Number	Treatment	Rank
(1)	Non-Dec	1.5
(2)	Non-Comp	1.5
(3)	Dec-VM	3.5
(4)	Dec-Reg	3.5

The level of significance on which the above ranking was based is only .25. As explained in Appendix I, the probability that all comparisons are true is at least .75.

Hypothesis 18: There are no significant differences for difference scores between the Posttest I and Posttest II STAN-Test 2 scores among the sixth grade students receiving the four treatments.

Accordingly, Table 102 exhibits the sums of squares, mean squares, and F-ratio for the difference scores of the second (Posttest I) and third rendition (Posttest II) of the test of arithmetic reasoning, STAN-Test 2.

Analysis of variance was used although the Hartley Maximum-F Test of homogeneity of variance showed the F-ratio to exceed slightly acceptable limits (Appendix H). Winer's (1962) assertion of the robustness of the test with respect to smaller deviations from the underlying assumptions led the researcher to carry through analysis of variance.

The relatively small F-ratio of Table 102 indicated that the treatment means are not very different in value. Data for Scheffe comparisons are shown in Table 103.

TABLE 102

ANALYSIS OF VARIANCE - ARITHMETIC REASONING DIFFERENCE SCORES - STAN-TEST 2 (Posttest II - Posttest I) - GRADE SIX (N=199)

Source	SS	df	Mean Square	F-Ratio
Among	136	3	45.50	3.196**
Within	2777	195	14.23	
Total	2913	198		

TABLE 103

SCHEFFE COMPARISONS BETWEEN MEANS FOR DATA OF TABLE 102

	Treatment Means						S.E.	
Comparison	2.1915	.3585	.0370	1.1333	$\sum ai^2$	d_i	of d_i	t
(1)vs(2)	1	-1	0	0	2	1.8330	.795	2.306#
(1)vs(3)	1	0	-1	0	2	2.1545		2.710*
(1)vs(4)	1	0	0	-1	2	1.0582		1.331
(2)vs(3)	0	1	-1	0	2	.3215		.404
(2)vs(4)	0	1	0	-1	2	-.7748		-.975
(3)vs(4)	0	0	1	-1	2	-1.0963		-1.379
(2)vs(1) +(3)+(4)	-1	3	-1	-1	12	-2.2863	1.947	-1.174
(1)vs(3) +(4)	2	0	-1	-1	6	3.2127	1.317	2.333#

Examination of Table 103 reveals relatively small differences among the treatment means of these difference scores. The approximate ranking is as follows:

Group Number	Treatment	Rank
(1)	Non-Dec	1
(4)	Dec-Reg	2
(2)	Non-Comp	3.5
(3)	Dec-VM	3.5

Hypothesis 20: There are no significant differences for scores on the sub-portion of STAN-Test 2 directly testing the concept of place value and numeration among sixth grade students receiving the four treatments.

In the test of arithmetic reasoning, STAN-Test 2, eight questions were selected for their relevance to the topic of place value and numeration. These questions formed a test referred to as the Place Value Subtest. Matching of the questions on the two test forms was exemplified by a sample question shown in Appendix J.

Because each child's responses to these questions were listed on his IBM data card, the reliabilities for this test could be computed. Both the Kuder-Richardson index of reliability and Spearman-Brown index are listed for each rendition of the Place Value Subtest in Table 8.

Tables 104, 105, and 106 show data for analyses of variance for the Place Value Subtests: Pretest, Posttest I, and Posttest II, respectively.

TABLE 104

ANALYSIS OF VARIANCE - PLACE VALUE SUBTEST (Pretest)
GRADE SIX (N=199)

Source	SS	df	Mean Square	F-Ratio
Among	16	3	5.432	1.962
Within	540	195	2.769	
Total	556	198		

TABLE 105

ANALYSIS OF VARIANCE - PLACE VALUE SUBTEST (Posttest I)
GRADE SIX (N=199)

Source	SS	df	Mean Square	F-Ratio
Among	12	3	4.097	1.036
Within	771	195	3.955	
Total	783	198		

TABLE 106

ANALYSIS OF VARIANCE - PLACE VALUE SUBTEST (Posttest II)
GRADE SIX (N=199)

Source	SS	df	Mean Square	F-ratio
Among	11	3	3.917	1.143
Within	668	195	3.427	
Total	679	198		

The non-significant F-ratios of Tables 104, 105, and 106 indicate non-rejection of the hypothesis of equal treatment means. There were no observable differences on these unadjusted means of scores on the Place Value Subtest given to sixth grade students.

Hypothesis 22: There are no significant correlations for sixth grade students separated according to sex and treatment among scores for intelligence, teacher judgment of arithmetic and reading ability, arithmetic computation, arithmetic reasoning, non-decimal numeration, and geometry.

Intercorrelations of scores and other data for sixth grade boys are shown in the lower left and for sixth grade girls in the upper right portions of Table 107. For almost all pairs of variables, the null hypothesis of no significant correlations, $H_o: \rho_{ij}=0$, was rejected.

Twenty-one of the 35 correlations shown for boys, were significant to the .01 level. Seventeen of the 35 correlations shown for girls were significant to the .01 level. Correlations for four pairs of variables did not match as to significance for the groups of sixth grade boys and sixth grade girls.

The four pairs of variables which did not match as to significance were further examined. For the difference between any two correlations to be significant to the .05 level for these two groups, its value must be approximately .17. Only one pair of variables showed correlations which differed by that amount for boys and girls, (11) and (28), Reading Estimate and Place Value (Posttest I). The analysis of Table 107 leads to the conclusion that correlations for sixth grade boys and girls are remarkably similar.

Table 107 reveals also that the place value posttest showed many more significant correlations with standardized tests of intelligence and arithmetic than did the non-decimal posttest or the geometry posttest. The geometry test showed no significant correlation with intelligence scores. The non-decimal test correlated only with the arithmetic computation. Except for this correlation, one may conclude that the non-decimal test and the geometry test were independent of standardized test scores. This suggests that in this sample, the successful learners of these two new topics were not the same children who were successful on more usual subjects. These two new topics may have provided success experiences for students of this sample not usually successful in routine topics of arithmetic.

Table 108 displays 27 correlations (of which 20 are significant to the .01 level) for the Non-Dec treatment group. Also shown are 27 correlations for the Non-Comp group of which 20 are significant.

Table 109 shows 21 correlations for the Dec-VM group, 15 of which are significant. Of the 21 Dec-Reg group correlations, 16 are significant to the .01 level.

The standardized tests and the teacher estimates show very high intercorrelations with each other. The non-decimal test shows high correlations with standardized tests but not with teachers' reading and arithmetic estimates. The geometry test correlates with both teacher estimates and standardized test scores. The similarity of intercorrelations for all four treatment groups is very great.

TABLE 107

INTERCORRELATIONS OF SCORES OF SIXTH GRADE STUDENTS
BOYS (N=128)[1] GIRLS (N=94)[2]

Variable[3]	(9)	(11)	(12)	(16)	(18)	(19)	(24)	(26)	(28)
(9) Age		-17	-10	-26	-12	-24	-03	-20	-09
(11) Rdg. Estimate (Tr.)	-02		73	69	55	63	24	-15	51
(12) Arith. Estimate (Tr.)	-20	72		51	55	71	23	-06	59 GIRLS
(16) CTMM	-46	48	52		48	64	22	07	48
(18) Stan. Arith. Comp. BOYS	-07	41	40	58		65	33	-02	67
(19) Stan. Arith. Reas.	-14	43	51	63	68		21	15	
(24) Non-Dec (Posttest I)	-05	08	04	28	38	26		-30	10
(26) Geometry (Posttest I)	-15	-10	05	12	19	31	-26		11
(28) Place Value (Posttest I)	-13	40	51	57	54		22	08	

[1] With an N=128, r must equal .22 to be significant at the .01 level.
[2] With an N=94, r must equal .26 to be significant at the .01 level.
[3] Numbered as in Appendix E.

TABLE 108

INTERCORRELATIONS OF SCORES FOR SIXTH GRADE STUDENTS
NON-DEC (N=47)[1] NON-COMP (N=53)[2]

Variable[3]	(9)	(11)	(12)	(16)	(18)	(19)	(24)	(26)	(28)	
(9) Age		14	12	-25	06	10		13	09	
(11) Rdg. Estimate (Tr.)	-25		60	39	40	44		52	52	
(12) Arith. Estimate (Tr.)	-30	86		42	48	58		63	63	NON-COMP
(16) CTMM (Pretest)	-55	47	50		47	44		57	57	
(18) Stan. Arith. Comp. (Pretest)	-15	52	47	62		57		47	57	
(19) Stan. Arith. Reas. (Pretest)	-33	49	53	75	69			71		
(24) Non-Dec (Posttest I)	-24	15	20	55	41	57				
(26) Geometry (Posttest I) NON-DEC									70	
(28) Place Value (Posttest I)	-16	31	41	52	53		47			

[1] With an N=47, r must equal .31 to be significant at the .01 level.
[2] With an N=53, r must equal .35 to be significant at the .01 level.
[3] Numbered as in Appendix E.

TABLE 109

INTERCORRELATIONS OF SCORES FOR SIXTH GRADE STUDENTS

DEC-VM (N=54)[1] DEC-REG (N=45)[2]

Variable[3]	(9)	(11)	(12)	(16)	(18)	(19)	(28)	
(9) Age		-36	-22	-57	-11	-37	-20	
(11) Rdg. Estimate (Tr.)	07		54	79	43	61	49	
(12) Arith. Estimate (Tr.)	-08	75		56	49	57	65	DEC-REG
(16) CTMM (Pretest)	-13	63	54		51	72	58	
DEC-VM (18) Stan. Arith. Comp. (Pretest)	-08	54	46	51		63	41	
(19) Stan. Arith. Reas. (Pretest)	-08	57	64	59	65		61	
(28) Place Value (Posttest I)	-07	59	61	52	52	77		

[1] With an N=54, r must equal .35 to be significant at the .01 level;
[2] With an N=45, r must equal .38 to be significant at the .01 level.
[3] Numbered as in Appendix E.

The degree of relationship indicated by the correlation coefficients for Tables 107, 108, and 109, according to Ferguson (1959), is as indicated below:

Table	Lower Boundary of Correlation Coefficient	Lower Boundary of Percent of Relationship
107	.222	4.9
	.263	6.9
108	.307	9.4
	.350	12.3
109	.346	12.0
	.376	14.1

These figures indicate the strength of certain relationships. No claims are made concerning causation. All percents are derived from correlation coefficients significant to the .01 level.

Hypothesis 24: There are no differences among sixth grade students' scores on the Non-Decimal Test (Posttest I) and the Non-Decimal Test (Posttest II).

Table 110 shows data which was used in the comparison of the scores on the two non-decimal tests by the Wilcoxon Matched-Pairs Signed-Rank Test.

The null hypothesis for this one-tailed analysis was as follows:

H_0: The scores of sixth grade students on the retention test of non-decimal systems are not significantly lower than scores on the first posttest.

The Wilcoxon Test examines whether the sum of the positive ranks is less than the sum of the negative ranks.

The lesser sum of the like ranks in this case would have had to be less than 271 for significance at the .05 level (N=39 non-zero differences).

The decision in this case was not to reject the null hypothesis. Sixth grade student performance on the retention test of non-decimal numeration could be considered equal to that on the first posttest.

TABLE 110

WILCOXON MATCHED PAIRS SIGNED RANK TEST - NON-DECIMAL TEST (Posttest I and Posttest II) GRADE SIX (N=47)

Score Differences	Rank of Negative Difference	Rank of Positive Difference
- 2	14	
- 1	4.5	
- 6	32.5	
- 4	25	
+ 1		4.5
+ 2		14
+ 5		29
- 7	36	
- 2	14	
+ 8		38
+ 1		4.5
- 2	14	
- 7	36	
- 1	4.5	
- 3	21	
+ 1		4.5
+ 4		25
- 4	25	
-13	39	
- 3	21	
- 7	36	
+ 1		4.5
- 2	14	
- 2	14	
+ 3		21
- 2	14	
+ 6		32.5
+ 1		4.5
+ 2		14
- 6	32.5	
+ 4		25
+ 2		14
- 5	29	
- 5	29	
+ 2		14
+ 4		25
- 1	4.5	
+ 2		14
+ 6		32.5
	459.5	320.5

CHAPTER V

COMPARISONS AMONG GRADES FOUR AND SIX

This chapter presents discussions of two hypotheses which will provide answers to two questions listed earlier. These questions are:

Will there be differences in test score intercorrelations among groups separated according to race and degree of advantage?

Which grade level will be more successful in learning non-decimal systems of numeration, grade four or grade six?

Hypothesis 25. There are no significant correlations for fourth and and sixth grade students separated according to race and level of advantage among scores for intelligence, teacher judgment of arithmetic and reading ability, arithmetic computation, arithmetic reasoning, non-decimal numeration, and intuitive geometry.

Table 111 shows intercorrelations of students separated according to membership in either Black or White race, as reported by classroom teachers.

Fifty-nine correlations are reported for Black students, 29 of which are significant to the .01 level. For the group of white students, 43 of the 59 correlations are significant to the .01 level

TABLE 111

INTERCORRELATIONS OF SCORES OF STUDENTS OF GRADES FOUR AND SIX

BLACK N=71[1] WHITE N=357[2]

Variable[3]	(9)	(11)	(12)	(16)	(18)	(19)	(20)	(21)	(24)	(25)	(26)	(28)
(9) Age		03	-09	-06	13	21	06	10	01	05	-05	11
(11) Rdg. Estimate (Tr)	12		65	59	44	49	50	53	17	23	10	48
(12) Arith. Estimate (Tr)	-10	61		48	41	45	44	48	09	17	00	47
(16) CTMM (Pretest)	12	63	49		48	58	46	63	22	26	06	54
(18) Stan.Arith.Comp. (Pretest)	39	31	27	56		67		60	26	27	05	51
(19) Stan.Arith.Reas. (Pretest)	36	50	30	59	48		58		20	27	15	WHITE
(20) Stan.Arith:Comp. (Posttest I)	25	52	45	55		47		61	17	19	-05	55
(21) Stan.ARith.Reas. (Posttest I)	19	44	39	59	54		60		18	23	14	
	BLACK											
(24) Non-Decimal (Posttest I)	00	00	00	03	00	01	00	02				-17
(25) Non-Decimal (Posttest II)	08	11	-17	09	05	-01	10	15				18
(26) Geometry (Posttest I)	43	18	02	37	40	52	13	27				09
(28) Place Value (Posttest I)	25	34	36	39	41		46		00	09	15	

1. With an N=71, r must equal .30 to be significant at the .01 level.
2. With an N=357, r must equal .13 to be significant at the .01 level.
3. Numbered as in Appendix E.

Many of the apparent differences in the level of significance between the two groups displayed in Table 111 occur with the variables (24) and (25) in correlation with the other variables. Because of the small number of Black students in the Non-Dec group, the correlation could not be reliably compared with correlations for the White students.

There are two pairs of correlations which were examined for significance of difference. The first pair involves the variables (12) and (18), arithmetic estimate and arithmetic computation. The second pair involves the variables (16) and (26), intelligence and geometry. In order for the difference between the correlations for these two groups to be significant to the .05 level, the correlation coefficients would have to differ by approximately .25. For the first pair the difference is not significant. The correlations of the second pair of variables differ significantly.

The interpretation of the significant difference for Black and White students on the correlation between intelligence and geometry may be of interest for planners of curriculum. For White students, the intelligence and geometry scores showed no significant correlation. For Black students, geometry scores and intelligence showed a highly significant correlation.

TABLE 112

INTERCORRELATIONS OF SCORES OF STUDENTS OF GRADES FOUR AND SIX

ADVANTAGED N=61[1] DISADVANTAGED N=42[2]

Variable[3]	(9)	(11)	(12)	(16)	(18)	(19)	(20)	(21)	(24)	(25)	(26)	(28)
(9) Age		21	-02	22	38	28	28	39	14	08	40	34
(11) Rdg. Estimate (Tr)	-21		54	54	22	37	37	36	05	-14	09	57
(12) Arith. Estimate (Tr)	-39	64		47	31	30	36	30	-09	-26	-05	36
(16) CTMM (Pretest)	-52	75	64		51	45	39	41	12	08	24	40
(18) Stan. Arith. Comp. (Pretest)	11	47	49	51		55		70	07	06	27	52
(19) Stan. Arith. Reas. (Pretest)	-09	60	62	67	68		57		05	-04	08	DISADV.
	ADVANTAGED											
(20) Stan. Arith. Comp. (Posttest I)	04	52	49	51		70		74	00	-14	09	71
(21) Stan. Arith. Reas. (Posttest I)	-26	65	62	73	60		61		-01	03	10	
(24) Non-Decimal (Posttest I)	-05	41	47	30	31	40	18	28				02
(25) Non-Decimal (Posttest II)	-12	44	53	32	27	34	12	27				-06
(26) Geometry (Posttest I)	-22	04	00	34	10	29	02	26				06
(28) Place Value (Posttest I)	-13	49	58	60	56		53		34	28	.24	

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1. With an N=61, r must be equal .33 to be significant at the .01 level.
2. With an N=42, r must be equal .39 to be significant at the .01 level.
3. Numbered as in Appendix E.

TABLE 113

INTERCORRELATIONS OF SCORES OF STUDENTS OF GRADES FOUR AND SIX

NORMAL GROUP N=327[1]

Variable[2]	(9)	(11)	(12)	(16)	(18)	(19)	(20)	(21)	(24)	(25)	(26)
(9) Age											
(11)Reading Estimate (Tr)	02										
(12)Arith. Estimate (Tr)	-10	62									
(16)CTMM (Pretest)	-02	52	40								
(18)Stan.Arith.Comp.(Pretest)	09	37	33	44							
(19)Stan.Arith.Reas.(Pretest)	23	44	37	54	63						
(20)Stan.Arith.Comp.(Posttest I)	02	45	38	42		50					
(21)Stan.Arith.Reas.(Posttest I)	11	46	42	59	56		57				
(24)Non-Decimal (Posttest I)	00	14	03	23	25	18	17	19			
(25)Non-Decimal (Posttest II)	07	22	10	27	27	27	22	24			
(26)Geometry (Posttest I)	00	-15	-02	00	06	15	09	10			
(28)Place Value (Posttest I)	10	39	39	47	44		49		14	16	04

1. With an N=327, r must be equal to .14 to be significant at the .01 level.
2. Numbered as in Appendix E.

Tables 112 and 113 display intercorrelations for students classified by their teachers as Advantaged, Disadvantaged, and Normal regarding educational opportunity. Of the 59 correlation coefficients displayed for each group, 34 are significant to the .01 level for the Advantaged group, 18 for the Disadvantaged group, and 41 for the Normal group.

Seventeen pairs of variables were tested for significant differences among the three pairings of advantage levels. The significant differences in correlation coefficients which resulted are shown in Table 114.

TABLE 114

SIGNIFICANT DIFFERENCES IN CORRELATION COEFFICIENTS
(.05 level)

Variable Pair	Adv.-Disadv.	Disadv.-Normal	Adv.-Normal
(11) (24)	Signif.	Non-Signif.	Signif.
(11) (25)	Signif.	Signif.	Non-Signif.
(12) (19)	Non-Signif.	Non-Signif.	Signif.
(12) (24)	Signif.	Non-Signif.	Signif.
(12) (25)	Signif.	Signif.	Signif.
(16) (19)	Non-Signif.	Non-Signif.	Non-Signif.
(16) (26)	Non-Signif.	Non-Signif.	Signif.
(19) (25)	Signif.	Non-Signif.	Non-Signif.

Five of the eight variable pairs of Table 114 evidenced significant differences between the groups classified as Advantaged and as Normal. Two significant differences were observed between the Disadvantaged and Normal groups. Five significant differences characterized the differences between the Advantaged and Disadvantaged groups.

Persistent differences among the correlations occur involving arithmetic estimate and arithmetic reasoning, intelligence and arithmetic reasoning, and intelligence and geometry.

The differences among students separated according to advantage level are far more numerous than those observed for groups distinguished by sex, treatment, or race. Attempts at explanation of these differences may prove to be fruitful areas for research.

Hypothesis 26. There are no differences among distribution of scores on the Non-Decimal Test (Posttest I and Posttest II) between the fourth and sixth grades.

The Kolmogorov-Smirnov Two-Sample Test was used to ascertain whether two independent samples have been drawn from the same population or populations having the same distribution. (See Appendix G)

Since the non-decimal test was administered twice, once as a posttest following the teaching of the unit and once as a retention test, the Kolmogorov-Smirnov Test was applied twice to the pertinent data. In this respect, Siegel (1956) states:

> The one-tailed test is used to decide whether or not the values of the population from which one of the samples was drawn are stochastically larger than the values of the population from which the other sample was drawn. (p.127)

Tables 115 and 116 display data used for the analysis of Non-Decimal Test (Posttest I) and Non-Decimal Test (Posttest II) respectively.

The null hypothesis in both cases was H: the fourth grade scores on the Non-Decimal test were as high as those of the sixth grade.

The null hypothesis in both cases was rejected in favor of the alternate hypothesis because of the highly significant computed Chi-Square value. The sixth grade sample must have been drawn from a population of higher score distribution than that from which the fourth grade sample was drawn. On both non-decimal tests therefore the sixth grade students as a group surpassed the fourth grade students.

TABLE 115

KOLMOGOROV-SMIRNOV TWO-SAMPLE TEST FOR NON-DECIMAL TEST (POSTTEST I)

Score Interval	Grade Four S43 (X)	Grade Six S47 (X)	Difference S43(X)-S47 (X)
40-41	1.0000	1.0000	0
38-39	.9070	.9787	-.0717
36-37	.8605	.8298	.0307
34-35	.8140	.5957	.2183
32-33	.7442	.5319	.2123
30-31	.6744	.4894	.1850
28-29	.6744	.4468	.2276
26-27	.6512	.3404	.3108
24-25	.6279	.2979	.3300*
22-23	.5814	.2766	.3048
20-21	.4884	.2340	.2544
18-19	.4615	.1915	.2736
16-17	.4186	.1064	.3122
14-15	.3488	.0638	.2850
12-13	.2791	.0638	.2153
10-11	.2093	.0426	.1667
8-9	.1628	.0426	.1202
6-7	.0465	0	.0465
4-5	.0232	0	.0232
2-3	0	0	0
0-1	0	0	0

* χ^2 for this max D, using the formula:

$$\chi^2 = 4D^2 \frac{n_1 n_2}{n_1 + n_2}$$ is 29.6413.

TABLE 116

KOLMOGOROV-SMIRNOV TWO-SAMPLE TEST FOR NON-DECIMAL TEST (POSTTEST II)

Score Interval	Grade Four S43 (X)	Grade Six S47 (X)	Difference S43(X)-S47(X)
40-41	1.0000	1.0000	0
38-39	.9302	.9787	-.0485
36-37	.8605	.8085	.0520
34-35	.7907	.6809	.1098
32-33	.7674	.5532	.2142
30-31	.6977	.4894	.2083
28-29	.5581	.4255	.1326
26-27	.5349	.3617	.1732
24-25	.4884	.2979	.1905
22-23	.4186	.2553	.1603
20-21	.3953	.1489	.2464
18-19	.3721	.0811	.2910*
16-17	.3023	.0426	.2597
14-15	.2558	.0426	.2132
12-13	.1628	.0213	.1415
10-11	.1163	.0213	.0950
8-9	.1163	.0213	.0950
6-7	.0930	0	.0930
4-5	.0232	0	.0232
2-3	.0232	0	.0232
0-1	0	0	0

* χ^2 for this max D, using the formula:

$\chi^2 = 4D^2 \frac{n_1 \; n_2}{n_1 + n_2}$ is 26.1382 .

CHAPTER VI

SUMMARY AND CONCLUSIONS

Summary

Four treatments in mathematics instruction were used with fourth and sixth grade students to evaluate the teaching of non-decimal systems of numeration in the elementary school.

Three testing periods allowed for administration of pretests, posttests, and retention tests of arithmetic computation, arithmetic reasoning, geometry, non-decimal numeration, and intelligence quotient. Each participating teacher supplied the following information for each student: age, sex, race, estimate of educational advantage, and estimate of reading and arithmetic levels.

The sample consisted of 430 students from eighteen classrooms. Nine grade four and nine grade six classes participated. Selection of these classes had been done on a random basis. Attempts were made to minimize teacher differences and mathematics background with a series of workshops and seminars held before and during the teaching period.

Data was analyzed statistically by testing twenty-six hypotheses: twelve each for fourth and sixth grade and two for comparisons involving both grade levels.

Results of testing these hypotheses allowed for discussion of the nine questions listed earlier in this study.

Conclusions

Effects of Four Treatments on Criterion Measures

Question One

Will the learning of non-decimal systems of numeration have any effect on scores of tests of computation and arithmetic reasoning given immediately after the teaching period and on those given several weeks later?

Posttest Computation Scores. There was no significant difference in posttest scores for any treatment group in either grade four or grade six. All treatment groups scored higher on the posttest of arithmetic computation than they had scored on the pretest. Grade six non-decimal group scores were slightly lower than scores attained by the other treatment groups. Lower posttest scores were also achieved by the non-computation groups in grades four and six.

Posttest scores of arithmetic computation were also analyzed with covariates of intelligence quotients and arithmetic computation pretest scores. Mean score increase for the non-decimal group of grade four was equal to that of the enriched decimal group. The regular decimal group, however, had scored the greatest gains. Grade six scores for the non-decimal group ranked last.

Score differences were studied separately by analysis of variance. Fourth grade non-decimal group was next to the last rank. The sixth grade non-decimal group scored last.

Scores on the posttest Stanford Achievement Computation Test achieved by both grade four and grade six non-decimal groups were not outstanding. Posttest group means were only slightly better than those achieved by the groups not studying any form of numerical computation. The study of non-decimal systems of numeration did not result in any significant mean score improvement in either grade four or grade six.

Posttest Arithmetic Reasoning Scores. The scores on the posttest of arithmetic reasoning were generally higher for all treatment groups. There were no significant differences among posttest means for all treatments at both grade levels.

Retention Test of Computation. Analyses of covariance were used on scores of the arithmetic computation retention test. There were no significant differences among the fourth grade treatment means. Those differences which had existed on the posttest mean scores were no longer present. Analysis of the difference scores showed that groups which had scored lowest on the posttest made the greatest gains following the experimental teaching period.

Analysis of the sixth grade retention test of arithmetic computation produced a different result. The mean of the group studying the regular decimal system program was significantly higher than the others. The enriched decimal group mean was the lowest of all four groups, suggesting that unfamiliar visual methods used for these students resulted in a minimal gain on the test of arithmetic computation.

Study of non-decimal systems did not advance significantly the computation scores of sixth grade students.

Retention Test of Arithmetic Reasoning. The mean of both the fourth and sixth grade treatment groups which studied non-decimal systems of numeration were the highest of all four treatment groups. This delayed positive transfer effect on the test of arithmetic reasoning was supportive of the beliefs of educators such as Rahmlow, 1965; Dutton, 1961; and Banks, 1961.

Effects on CTMM Scores. Some educators have stated that intelligence tests resemble achievement tests (Davis, 1960). An examination of the California Test of Mental Maturity revealed that two of its seven subtests were tests of arithmetic computation and problem solving. Other subtests included questions based on perception of geometric figures and knowledge of some quantitative relationships. Therefore, analysis was made of the second rendition of the CTMM using the first rendition, the pretest, as covariate.

Fourth grade results showed that all groups had equal means except the enriched decimal group. This mean was significantly lower than the other three.

Sixth grade results indicated that the non-computational group (intuitive geometry) mean was higher than those of the other three groups. Non-decimal treatment scores did not affect CTMM results of either grade.

Question Two

Will the teaching of a non-numerical topic such as intuitive geometry affect scores on standardized tests of arithmetic achievement and reasoning?

The non-computational group of grade four suffered temporary losses on the standardized tests. These losses apparently were made up during the seven weeks between the post- and retention tests.

On the posttest and retention test following the teaching period, the non-computation treatment group of grade six scored as well as the non-decimal group. No significant changes were noted.

Question Three

Will the enriching of the regular arithmetic program with visual devices and nontextual materials affect scores on standardized tests of arithmetic computation and reasoning?

The enriched decimal program was not effective for either the fourth or sixth grade students. Arithmetic achievement test scores were significantly lower on the posttest for both fourth and sixth grades.

Question Four

Will the teaching of the usual arithmetic program of decimal numeration affect scores on standardized tests of arithmetic achievement and reasoning?

The regular decimal treatment mean was highest on the posttest and retention test of arithmetic reasoning. According to these criteria, this treatment was the most successful for both fourth and sixth grades

Effects of Study of Non-Decimal Systems

Question Five

Will the students who learned non-decimal systems of numeration retain this ability over a period of time?

Pretest and retention test scores of the non-decimal test were analyzed separately for each grade by the Wilcoxon Matched-Pair Signed-Rank Test. The fourth grade retention test scores were lower than the posttest scores, indicating a loss of this specialized knowledge during the weeks following the teaching of the unit.

Retention test scores for sixth grade students were almost equal to the posttest scores.

Briefly, fourth graders did not retain their ability to compute in non-decimal numeration. Sixth graders did retain this ability as measured by the test of non-decimal systems of numeration.

Question Six

Will the learning of non-decimal systems have any effect on the scores of that portion of the arithmetic reasoning test containing questions on place value and numeration?

The fourth grade non-decimal treatment group mean on the Place Value Subtest was tied for second place with the non-computational group on all three tests (pretest, posttest, and retention test). No differences were noted among any of the four grade six treatment groups on scores of any of the three renditions of the Place Value Subtest (pretest, posttest, and retention test).

The learning of non-decimal systems of numeration did not add or detract from students' ability to answer questions concerning place value and numeration in the decimal system.

Question Seven

Will significant positive correlations result on scores on the non-decimal test and scores on the arithmetic computation and reasoning tests? Were the same students successful on both tests?

Analysis of correlations among these test scores reveals that for both fourth and sixth grade samples, the same students were successful on the arithmetic computation and reasoning pretests and on the non-decimal test immediately following the teaching period.

Examination of the correlations for grade four boys and girls discloses that when arithmetic computation is correlated with non-decimal test, the coefficients are not significant for boys or girls. The correlation for boys however, was fiarly close to the .01 level used in this study.

Further inspection of separate correlation tables for sixth grade boys and girls reveals that the arithmetic reasoning and non-decimal test correlation coefficient for the girls failed by a small amount to be significant to the .01 level.

Generally, the same students were successful on the standardized arithmetic computation and reasoning tests and on the non-decimal test.

Question Eight

Which grade level will be more successful in learning non-decimal systems of numeration, grade four or grade six?

Results of the posttest and retention tests were analyzed by the Kolmogorov-Smirnov Two-Sample Test. The analysis showed that on both tests the grade six sample distribution was higher than that of the grade four sample.

Many questions on the tests for the two grade levels were identical. Additional questions were prepared for grade four and grade six. Test questions for each grade were item analyzed. Fourth grade student retention test scores were generally lower than their posttest scores. Scores for the sixth graders on both tests were not significantly different.

Sixth grade students of this study were generally more successful in learning and retaining their knowledge of non-decimal systems of numeration than were fourth grade students.

Intercorrelation Among Groups

Question Nine

Will there be differences in test score intercorrelations among groups separated according to:

a. treatment group
b. sex
c. race
d. degree of advantage?

Treatment Group

The grade four Dec-VM group, which had the lowest pretest mean of intelligence, displayed differences among intercorrelations from those of the other three groups. Standardized scores for this portion of the fourth grade sample might be described as erratic and unreliable. The intercorrelations therefore would not follow patterns found in analyses of the other three groups.

The intercorrelations for the grade six treatment groups were very similar. Differences among them were not significant.

Sex

Intercorrelations among scores of fourth grade boys and girls were very similar.

Scores of girls in the sixth grade differed slightly from those of the boys in that grade. For both groups, however, geometry scores were independent of standardized test and intelligence test scores.

Race

Significant differences among intercorrelations of students separated according to race were few in number. These differences generally involved teacher estimates of reading, teacher estimates of arithmetic ability, and geometry scores.

Degree of Advantage

When students were separated into groups according to teacher estimate of educational advantage, a number of differences were observed. Many differences occurred between the Advantaged and Normal groups. These differences usually involved correlation with teacher estimates of reading and arithmetic. The arithmetic reasoning test showed a much higher correlation with teachers' estimate of arithmetic for the Advantaged group than it did for the Normal group. For the Advantaged group, the geometry and intelligence test scores showed positive correlation, for the Normal group none. For the Advantaged group, teachers' arithmetic estimates showed positive correlation with scores on the non-decimal tests. The teachers' estimates of arithmetic ability were independent of scores on the non-decimal tests for the Normal group.

Normal and Disadvantaged group correlations evidenced two significant differences. Both of these involved scores on the non-decimal retention test. The Normal group evidenced significant correlations between reading estimate and non-decimal test and no correlation between arithmetic estimate and non-decimal test. The Disadvantaged group showed negative correlations for both pairs of variables of significantly lower magnitude than the correlation coefficients for the Normal group.

Teacher judgments were the direct opposite of students' performance on the non-decimal test criterion for the Disadvantaged group.

Most differences between the Advantaged and Disadvantaged groups on intercorrelations involved teacher estimates which generally showed much higher correlations with standardized test scores for the Advantaged than for the Disadvantaged group.

Summary of Intercorrelation Analysis

Differences in intercorrelation because of race and treatment were negligible. Differences because of sex were slight. Differences related to educational advantage were numerous and involved all pairings of the three designations of levels of advantage.

Transfer Analysis

This report described an experiment in the teaching of non-decimal numeration and its effect upon decimal computation. Certain conditions associated with positive transfer effect were designed into the experiment. These were:

1. A substantial period of time was devoted to the exclusive teaching of the assigned mathematics program. Comparison with other studies testing similar hypotheses indicated that the instructional period of this study (5-6 weeks of at least one hour a day) surpassed the instructional time allotted by Lerch, 1963; McCormick, 1965; Schlinsog, 1965; and Jackson, 1965. All of these researchers recommended a more extended time period of instruction in the topic.

2. Use of materials and methods appropriate for each teacher was accomplished with the assistance of the investigator. There was no indication in other studies dealing with non-decimal systems in which teacher workshops had been utilized. Jackson (1965) supplied teachers with a teacher's manual after two meetings with them. Teacher familiarity with classroom materials is recognized as an important aspect of teaching. Investigator-prepared worksheets and units are important for instructional purposes, but teacher-developed materials, as in this study, may have more impact.

3. Cooperating teachers maintained a favorable attitude throughout the study. Each teacher completed his assignment without any pressure or anxiety. An amicable relationship between the investigator and cooperating teachers was maintained throughout.

4. The four treatments carried out in this study were the only mathematics topics taught during the experimental period.

Statistical analysis of scores on pretest, posttest, and retention tests of arithmetic computation and reasoning suggested certain conclusions concerning the transfer value of the study of non-decimal systems. Normal score rise as a result of students' maturational growth as well as increasing familiarity with a given test may be expected with subsequent administrations of the same test. Covariance analysis of the posttest and retention tests of arithmetic computation and reasoning, however, disclosed a significantly higher retention test score on the test of arithmetic reasoning for both grade levels of students in the Non-Dec group.

This positive transfer effect may have resulted from the teachers' attention to methods favoring such transfer. Some principles known to favor certain types of positive transfer and which may be operating here are the following:

1. Emphasis on meaningfulness. Principles of base and place were stressed throughout the teaching period. A variety of student activities was available for illustration of each concept.

2. Differentiation of stimuli. This was established by teachers' constant use of subscript notation as in 101_{five} to mean 101 base five and not base ten.

3. Positive attitude of the teachers. This attitude appeared to be the result of teachers' voluntary participation in teacher workshops. School administrators encouraged teacher efforts and evidenced complete cooperation with this investigator.

4. Positive attitude of students. Interest in the new topic was high. Success experiences had been planned by teachers with emphasis on practice and participation.

Implications for the Curriculum of the Elementary School

The carefully-planned-for inclusion of this topic into the elementary school curriculum is recommended. Rote learning methods which do not stress meaning and place value are to be discouraged. Mere imitation of current trends should also be avoided. If retention of learning is one objective in teaching non-decimal systems, the topic should be postponed until the upper elementary grades.

Recommendations for future research therefore will consider the importance of this topic to the elementary school curriculum. Three areas are suggested: grade-placement of the topic, methods of teaching, and the long-range effects involved with learning of non-decimal systems.

Some recent programs have demonstrated that it is possible to teach base generalization at the primary level before grade three and follow this by specialization in base ten. Clarification of this issue through research is possibly more important than deciding whether to teach the topic in grade five or six.

Further research in methods of teaching non-decimal numeration should explore a variety of ways to present numerals and number names which act as stimuli for arithmetic operations.

For instance, is it better to use students' own nonsense syllables, such as "mic, mac, moe" for "one, two, three" in their creation of new numeration systems?

Similarly, should new symbols be invented such as "*, #, &" for "one, two, three?"

Further study of the effectiveness of developing story lines in teaching non-decimal systems is needed as well.

The teaching of non-decimal systems of numeration, using readily available textbooks for the upper elementary grades, should be undertaken only if affirmative answers may be given the following questions:

1. Do teachers know the content well?

2. Are teachers knowledgeable about and willing to use materials and methods to supplement and enhance those suggested by students' textbooks?

3. Are teachers able to devote adequate time to the topic?

4. Are teachers aware of those topics which when taught simultaneously might interfere with the desired learning?

5. Will the teaching of non-decimal systems with emphasis on base and place-value be followed by teaching the decimal system in a similar manner?

6. Does the teacher make the learning of the topic as meaningful as possible so students may culminate this learning with success and favorable attitudes?

Study of the long-range effects of learning non-decimal numeration, with and without interim reinforcement would be a desirable research project. Comparison between the teaching of the topic on several occasions over a period of years might be made with a concentrated teaching period in the upper elementary grades.

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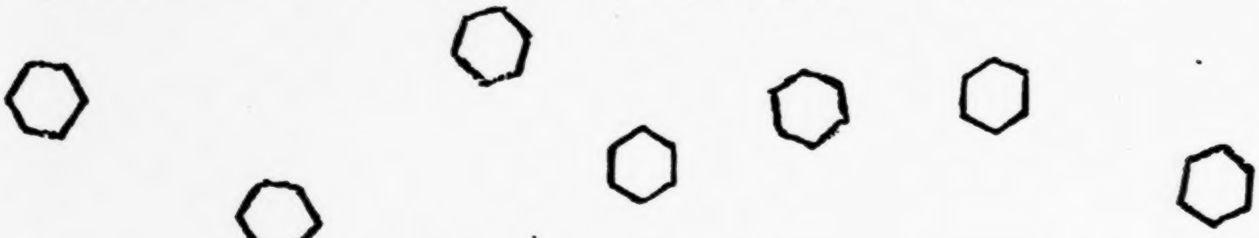
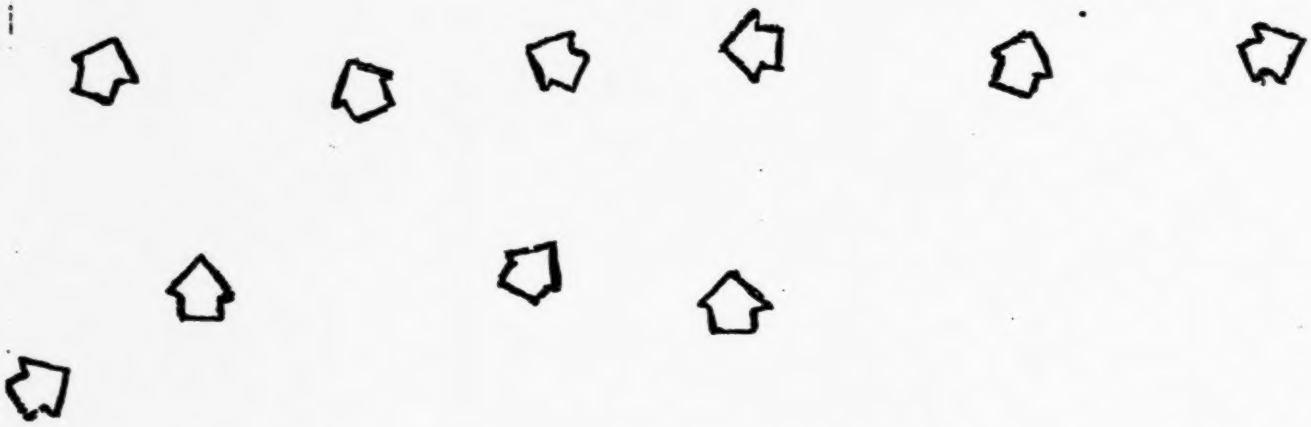
APPENDIX A

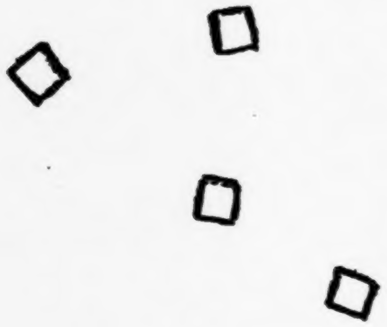
WORKSHEET SAMPLES OF NON-DECIMAL TREATMENT



___ hexagons



_____ arrowheads



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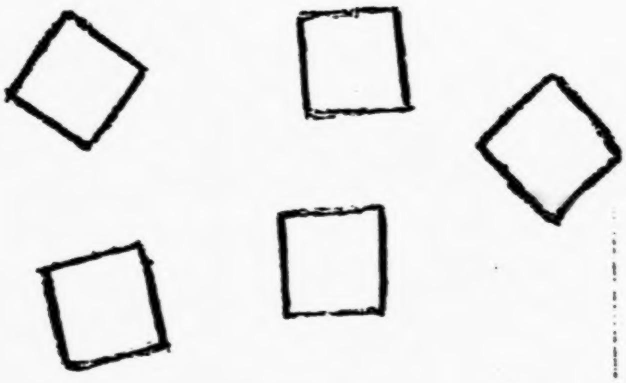
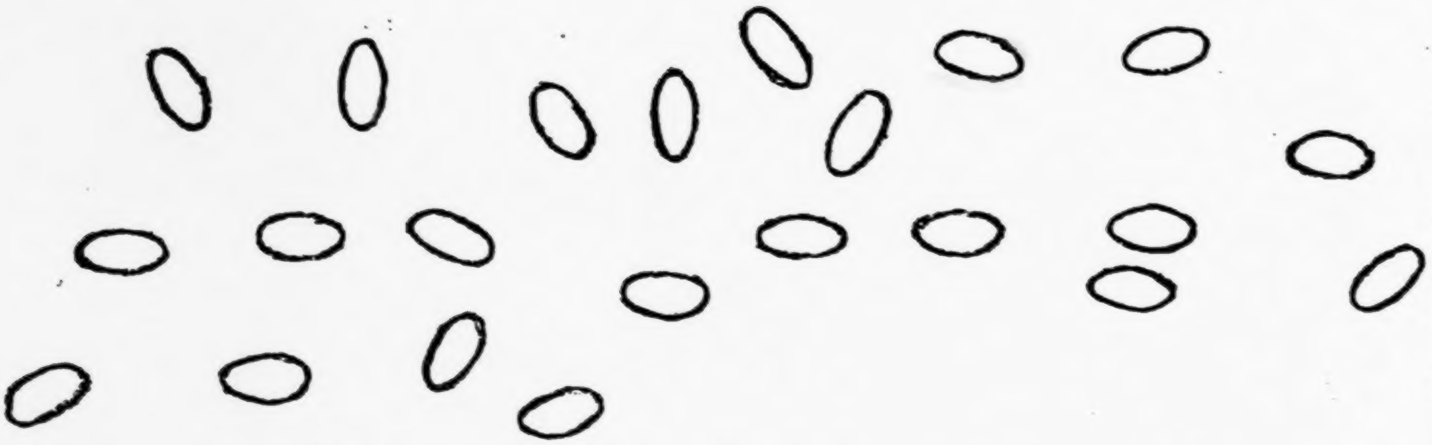


____ large squares



_____ eggs

Numerals

$12_{five} =$ ______

$23_{five} =$ ______

$4_{five} =$ ______

$2_{five} =$ ______

$10_{five} =$ ______

$32_{five} =$ ______

$34_{five} =$ ______

$44_{five} =$ ______

$100_{five} =$ ______

$111_{five} =$ ______

$122_{five} =$ ______

$312_{five} =$ ______

$302_{five} =$ ______

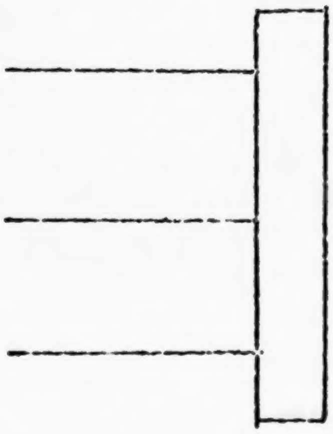
$300_{five} =$ ______

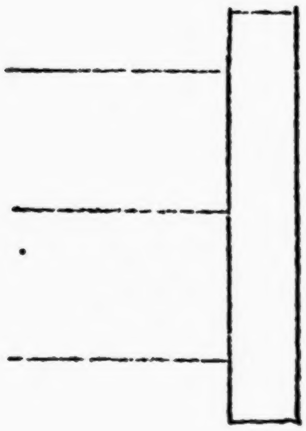
$40_{five} =$ ______

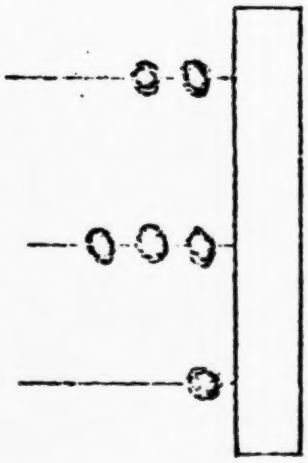
$421_{five} =$ ______

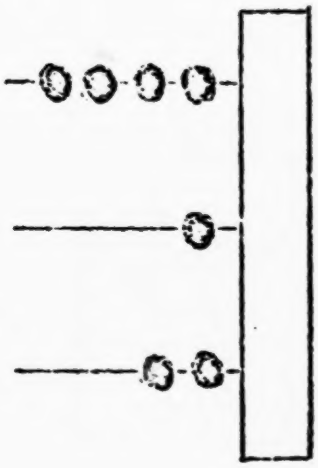
$42_{five} =$ ______

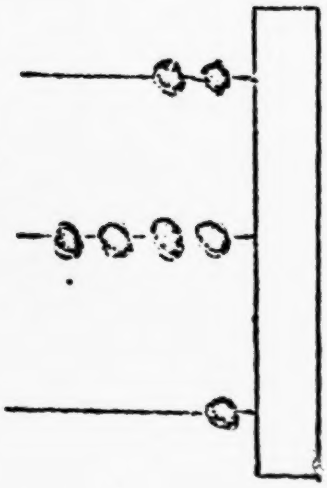
$41_{five} =$ ______

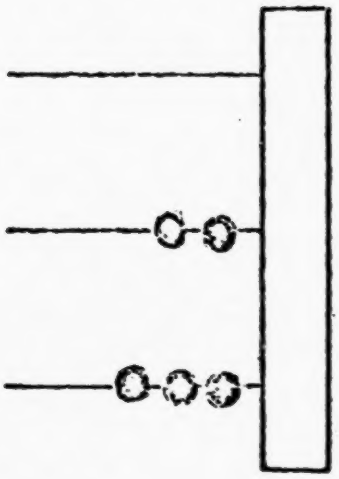












$$\begin{array}{r} 41_{\text{five}} \\ -\ 12_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 30_{\text{five}} \\ -\ 11_{\text{five}} \\ \hline \end{array}$$

$$\begin{array}{r} 40_{\text{five}} \\ -\ 4_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 31_{\text{five}} \\ -\ 12_{\text{five}} \\ \hline \end{array}$$

$$\begin{array}{r} 100_{\text{five}} \\ -\ 20_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 100_{\text{five}} \\ -\ 40_{\text{five}} \\ \hline \end{array}$$

$$\begin{array}{r} 100_{\text{five}} \\ -\ 10_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 100_{\text{five}} \\ -\ 4_{\text{five}} \\ \hline \end{array}$$

$$\begin{array}{r} 100_{\text{five}} \\ -\ 2_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 100_{\text{five}} \\ -\ 1_{\text{five}} \\ \hline \end{array}$$

$$\begin{array}{r} 100_{\text{five}} \\ -\ 11_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 100_{\text{five}} \\ -\ 12_{\text{five}} \\ \hline \end{array}$$

$20_{\text{five}} - 13_{\text{five}} =$ ______

$21_{\text{five}} - 4_{\text{five}} =$ ______

$30_{\text{five}} - 12_{\text{five}} =$ ______

$41_{\text{five}} - 23_{\text{five}} =$ ______

$40_{\text{five}} - 31_{\text{five}} =$ ______

$100_{\text{five}} - 30_{\text{five}} =$ ______

$101_{\text{five}} - 30_{\text{five}} =$ ______

$101_{\text{five}} - 40_{\text{five}} =$ ______

$101_{\text{five}} - 10_{\text{five}} =$ ______

$102_{\text{five}} - 10_{\text{five}} =$ ______

$104_{\text{five}} - 20_{\text{five}} =$ ______

$103_{\text{five}} - 21_{\text{five}} =$ ______

$102_{\text{five}} - 41_{\text{five}} =$ ______

$102_{\text{five}} - 42_{\text{five}} =$ ______

$102_{\text{five}} - 43_{\text{five}} =$ ______

$102_{\text{five}} - 44_{\text{five}} =$ ______

$102_{\text{five}} - 100_{\text{five}} =$ ______

APPENDIX B

DEVELOPMENT OF NON-DECIMAL TESTS

Development of Test: Non-Decimal Numeration

Nature of the Questions

Sixty-four questions developed by the researcher were similar to those used on students' worksheets. Coverage of subject matter was obtained by use of a test planning grid (Table 117).

These questions were reviewed by teachers who taught that unit. Suggestions and changes were requested on a form developed for the purpose.

A test of seventy-two questions was constructed based on the teacher review for each grade level (Tables 118 and 119).

Two test-constructor classes were chosen according to the following criteria:

1. Large ranges existed in the intelligence quotient scores and in the arithmetic achievement scores.

2. The teachers' methods were judged by the researcher to be fairly typical of the treatment group.

After the test was administered to each test-constructor class, each question was analyzed for difficulty and discrimination.

Difficulty of Items on the Non-Decimal Test

Questions were selected for the Non-Decimal Test from a difficulty range of 30% to 70% with preference given to the 50% level. Table 120 lists the difficulty of items on the grade four test; Table 121 lists those developed for grade six.

TABLE 117

Grid for Tryout Questions of Non-Decimal Test Submitted to Teachers

	By Grouping	By Counting	By Diagram	By Abacus	By Table	Standard Notation
Meaning of Base Ten Numerals	0	2	0	0	0	2
Meaning of Base Five Numerals	2	3	2	2	0	1
Conversion from Base Five to Base Ten	0	0	1	2	1	4
Conversion from Base Ten to Base Five	2	0	1	1	1	2
Addition in Base Five Without Regrouping	1	1	1	1	1	1
Addition in Base Five With Regrouping	1	1	1	1	1	1
Subtraction in Base Five Without Regrouping	1	1	1	1	1	1
Subtraction in Base Five With Regrouping	1	1	1	1	1	1
Multiplication in Base Five Without Regrouping	1	0	1	1	1	1
Multiplication in Base Five With Regrouping	1	0	1	1	1	1

TABLE 118

Grid for Fourth Grade Tryout Test on Non-Decimal Systems

	By Grouping	By Counting	Dot Diagram	By Abacus	By Table	Standard Notation
Meaning of Base Ten Numerals	0	2	0	0	0	2
Meaning of Base Five Numerals	2	3	2	2	0	1
Conversion from Base Five to Base Ten	0	0	1	2	1	3
Conversion from Base Ten to Base Five	2	0	1	1	1	2
Addition in Base Five Without Regrouping	2	1	1	1	1	2
Addition in Base Five With Regrouping	2	0	2	1	2	3
Subtraction in Base Five Without Regrouping	1	2	1	2	1	2
Subtraction in Base Five With Regrouping	1	1	1	1	1	4
Multiplication in Base Five Without Regrouping	1	0	1	1	0	0
Multiplication in Base Five With Regrouping	1	0	1	1	1	0

TABLE 119

Grid for Sixth Grade Tryout Test on Non-Decimal Systems

	By Grouping	By Counting	Dot Diagram	By Abacus	By Table	Standard Notation
Meaning of Base Ten Numerals	0	2	0	0	0	2
Meaning of Base Five Numerals	2	3	2	2	0	1
Conversion from Base Five to Base Ten	0	0	1	2	2	3
Conversion from Base Ten to Base Five	2	0	1	1	1	2
Addition in Base Five Without Regrouping	1	1	1	1	1	1
Addition in Base Five With Regrouping	1	1	1	1	1	1
Subtraction in Base Five Without Regrouping	1	1	1	1	1	1
Subtraction in Base Five With Regrouping	1	1	1	1	1	1
Multiplication in Base Five Without Regrouping	1	0	1	1	1	1
Multiplication in Base Five With Regrouping	1	0	1	1	1	1

Discrimination of Items on Non-Decimal Test

Because of the small number of students in each test-constructor class, the upper and lower halves, rather than the upper and lower 27% were used to determine the discriminating power of each item.[1] Correlation between each item and total score was determined according to the method of Mosier & McQuitty (1940) who suggested that determination be made of the percent correct in each half of the test subjects and subsequent use of an abac designed for the purpose of determining the correlation r. (Table 122)

Difficulty and discrimination of items as well as content coverage in relation to the test grid, were considered in the final choice of test items; a procedure suggested by Cox (1964) (Tables 120 and 121).

---

[1] Frederick B. Davis, "Item selection techniques," In E.F. Lindquist (Ed.), Educational Measurement, Washington, D.C.: American Council on Education, 1951, Pp. 266-328. Davis pointed out that..."the loss of reliability incurred by estimating indices from only 54 percent of the sample tested is not sufficient to be of practical consequence when the two criterion groups employed include at least 100 examinees apiece (p.283)."

TABLE 120

Degree of Difficulty and Discrimination of Items on Fourth Grade Test Constructor Group - Non-Dec

Item	Diff	Discr.	Item	Diff.	Discr.
2	.87	.20	38	.46	.38
3	.54	.62	39	.58	.54
4	.16	.43	40	.67	.30
5	.29	.15	41	.92	.00
6	.71	.74	42	.83	.50
7	.75	.30	43	.33	.55
8	.38	.40	44	.88	.00
9	.46	.80	45	.50	.53
10	.58	.90	46	.95	.00
11	.33	1.00	47	.54	.10
12	.54	.63	48	.92	.00
13	.72	.75	49	.54	.80
14	.50	.85	50	.50	.72
15	.39	.40	51	.58	.28
16	.89	.00	52	.33	.27
17	.58	.50	53	.92	.00
18	.79	.70	54	.42	.25
19	.39	.40	55	.38	.85
20	.50	.89	56	.92	.00
21	.71	.45	57	.67	.55
22	.50	.50	58	.50	.72
23	.17	.95	59	.58	.28
24	.08	.00	60	.67	.30
25	.39	.90	61	.21	.60
26	.92	.10	62	.67	.20
27	.50	.85	63	.54	.80
28	.42	.90	64	.42	.50
29	.71	.95	65	.63	.20
30	.59	.50	66	.38	.64
31	.54	.50	67	.50	.00
32	.50	.86	68	.54	.80
33	.88	.80	69	.67	.83
34	.58	.50	70	.67	.53
35	.55	.80	71	.63	.65
36	.75	.68	72	.58	.52
37	.42	.90			

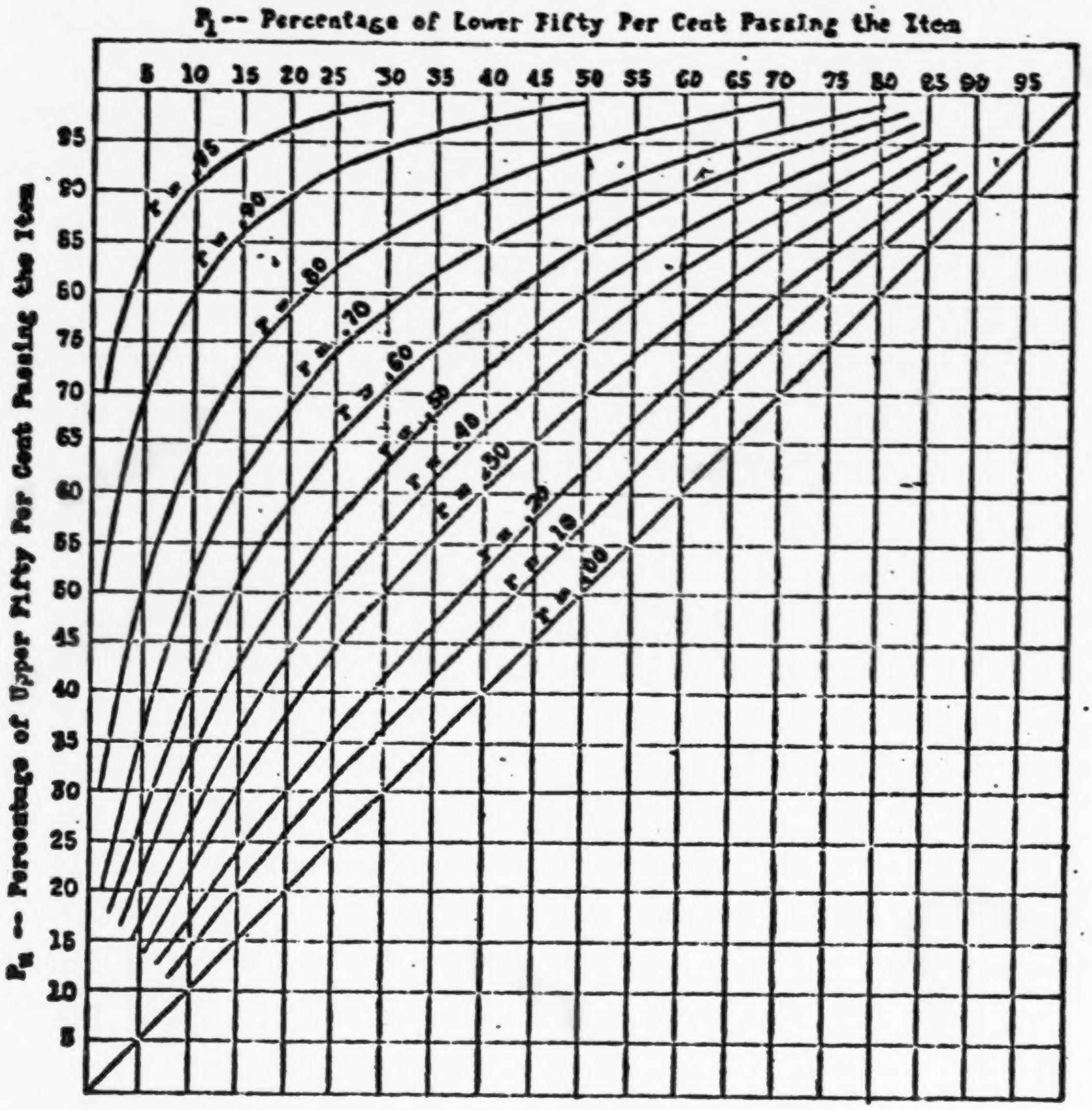
TABLE 121

Degree of Difficulty and Discrimination of Items on Sixth Grade Test Constructor Group - Non-Dec

Item	Diff.	Discr.	Item	Diff.	Discr.
2	.66	.00	38	.54	.93
3	.77	.90	39	.77	.30
4	.38	.50	40	.77	.61
5	.62	.86	41	.00	.00
6	.92	.00	42	.81	.85
7	.92	.00	43	.65	.85
8	.85	.60	44	.65	.64
9	.73	.93	45	.65	.95
10	.73	.73	46	.92	.10
11	.35	.60	47	.84	.20
12	.92	.20	48	.84	.20
13	.88	.10	49	.73	.90
14	.85	.80	50	.65	.90
15	.88	.10	51	.77	.60
16	.92	.15	52	.73	.92
17	.42	.75	53	.77	.90
18	.88	.20	54	.65	.64
19	.88	.20	55	.62	.60
20	.62	.52	56	.88	.15
21	.69	.95	57	.88	.12
22	.65	.40	58	.78	.22
23	.65	.45	59	.65	.64
24	.80	.60	60	.78	.24
25	.54	.68	61	.77	.63
26	.89	.00	62	.78	.22
27	.77	.70	63	.62	.86
28	.65	.90	64	.68	.25
29	.88	.20	65	1.00	.00
30	.80	.85	66	.73	.72
31	.80	.20	67	.72	.22
32	.78	.25	68	.73	.90
33	.92	.10	69	.96	.04
34	.62	.73	70	.92	.12
35	.88	.10	71	.42	-.60
36	.96	.00	72	.70	.21
37	.69	.54			

TABLE 122

MOSIER-MCQUITTY ABACS FOR ITEM DISCRIMINATION



Abacs for Item-Test Correlation from Percentage of Upper and Lower Fifty Per Cent Passing the Item

The test as constructed resulted in large ranges of scores on both administrations. Kuder-Richardson$_{20}$ reliability is shown in Table 123.

TABLE 123

Kuder-Richardson 20 Reliability Non-Decimal Test r_{tt}

Grade	Class	Posttest I	Posttest II
4	1	.908	.840
4	2	.942	.896
6	1	.937	.865
6	2	.882	.881

Kelly (1927) required a minimum reliability of .90 to evaluate differences in level of group accomplishment in two or more performances and .94 to evaluate level of individual accomplishment.[1] The reliabilities in Table 123 are near Kelly's requirement and resulted from careful attention to test construction requirements on the part of the researcher. (See Epstein, 1968 on this aspect of test construction).

Organization of Test

Posttest I. The Non-Decimal Numeration Test was organized as a multiple-choice test with four choices given for each item. An attempt was made to eliminate or reword those choices or distractors which were selected by no one in the trial of the test (Ebel, 1951).

Posttest II. The same forty questions in different order constituted the Non-Decimal Numeration Test administered during Posttest II. In both administrations of the test, teachers were instructed to permit everyone to complete the test. Average time for completion was one hour.

---

[1] Kelley, T.L. Interpretation of educational measurements. Yonkers, N.Y.: World Book, 1927., as quoted by Thorndike, R.L. "Reliability." in E.F. Lindquist (Ed.) Educational measurement. Washington, D.C.: American Council on Education, 1951. Pp. 560-620.

Directions for Administration of Test
on Non-Decimal Numeration

This is not a power test to show the efficiency of your teaching or the ability of your class as a whole.

This is a separator test to distinguish the strong learner of this subject from the others.

Have the children write their names in three places:
1. Answer card front
2. Answer card back
3. Upper right-hand corner of Direction Page

READ DIRECTIONS ALOUD WITH STUDENTS

This is not a speed test. Allow sufficient time for all to complete (within reasonable limits). To keep movement in the room during the test period down to a minimum allow children who have completed before the papers are picked up to read some other materials at their seats, but not to move about.

Children may write on question pages and do not need scratch paper.

Supervise carefully to see that each child is doing his own work--and is really working on these questions, not merely filling in a pretty design on the answer sheet.

If children ask questions about the numerals or the words, you may read these to them at their seats individually. Do not answer concept-type questions. If a child should ask such a question, tell him to use his best judgment or to choose the best answer from the group (in his opinion).

APPENDIX C

NON-DECIMAL TEST - GRADE FOUR

NON DECIMAL TEST GRADE FOUR

Read directions carefully.

This is a multiple choice test.

For each question, only ONE answer is correct.

Read each question carefully. Do your figuring on the question sheet. Then select the right answer from the choices given.

Find the number of the question
on the answer card.

Circle the letter which goes with
the answer you have chosen.

SAMPLE

1. How much is $2_{five} + 2_{five}$?

   a. 3_{five}
   b. 4_{five}
   c. 11_{five}
   d. 12_{five}

The correct answer is b. Look on your answer card to see the circle around b.

1. a (b) c d

When told to do so, begin to work each question, marking your answer sheet as you go along. Take your time, work carefully, and try not to make wild guesses.

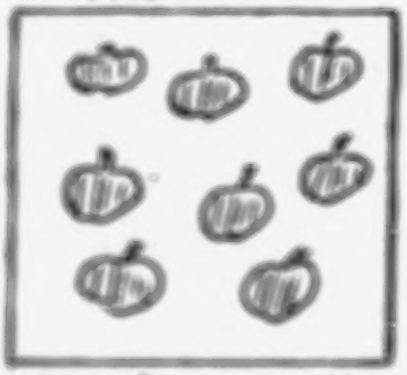
Some easy questions come after some hard questions.

Do not leave your seat. If you have any questions, raise your hand and the teacher will come to your seat.

When you finish each page, go on to the next.
DO NOT TURN BACK.

When you reach the end of the test, turn the test booklet over on your desk. Cover the answer sheet and sit quietly.

2) How many more apples should be put into the box to make 24_{five} ?



a. 3_{five}
b. 4_{five}
c. 10_{five}
d. 11_{five}

3) Which base five number is 1 more than 44_{five} ?

a. 45_{five}
b. 54_{five}
c. 100_{five}
d. 104_{five}

4) Which of the following is the correct choice for the missing number in $3682 = 3000 + 600 + \_\_\_ + 2$

a. 8
b. 68
c. 80
d. 82

5) Which base ten (decimal) numeral has the same value as 110_{five} ?

a. 26
b. 30
c. 35
d. 55

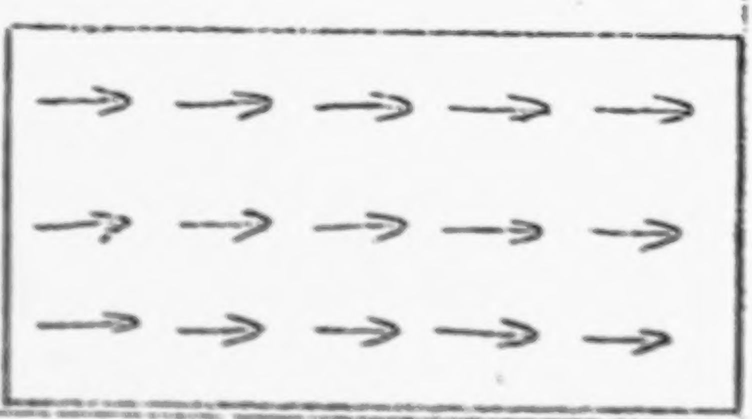
6) Which base five numeral is correct for the number of arrows in the picture?

a. 15_{FIVE}

b. 25_{FIVE}

c. 30_{FIVE}

d. 31_{FIVE}



7)

$$\begin{array}{r} 304_{\text{FIVE}} \\ -\ 140_{\text{FIVE}} \\ \hline \end{array}$$

a. 14_{FIVE}

b. 24_{FIVE}

c. 114_{FIVE}

d. 214_{FIVE}

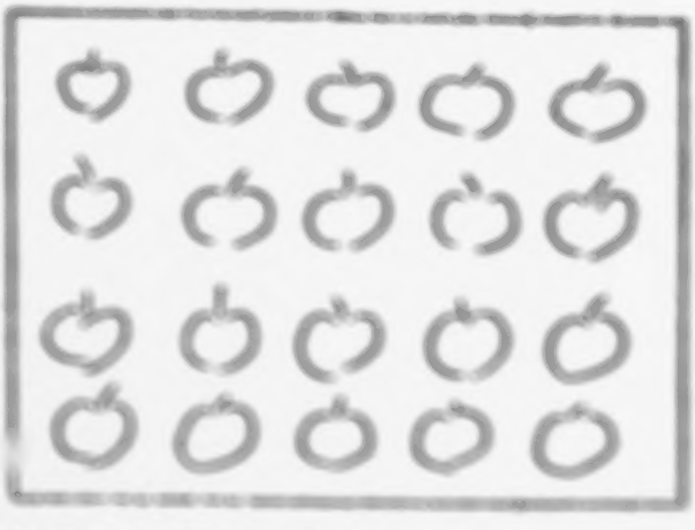
8) If 10_{FIVE} more apples were put in the box, how many would be inside?

a. 44_{FIVE}

b. 45_{FIVE}

c. 55_{FIVE}

d. 100_{FIVE}



9) $$\begin{array}{r} 123_{\text{FIVE}} \\ +\ 241_{\text{FIVE}} \\ \hline \end{array}$$

a. 364_{FIVE}

b. 404_{FIVE}

c. 414_{FIVE}

d. 424_{FIVE}

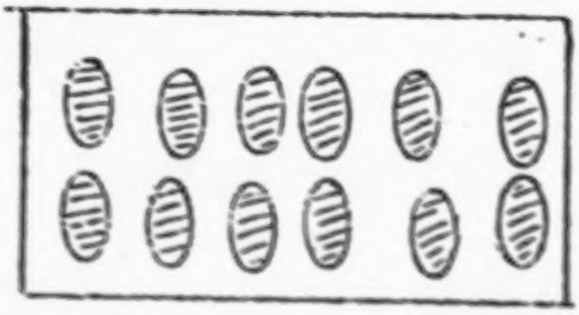
10) If three eggs are removed from the group of eggs in the box, how many are left?

a. 9_{FIVE}

b. 14_{FIVE}

c. 20_{FIVE}

d. 21_{FIVE}



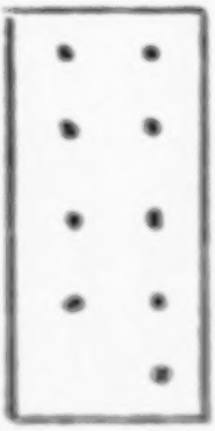
11) Which base five numeral is correct for the number of dots shown in the diagram?

a. 9_{FIVE}

b. 14_{FIVE}

c. 41_{FIVE}

d. 45_{FIVE}



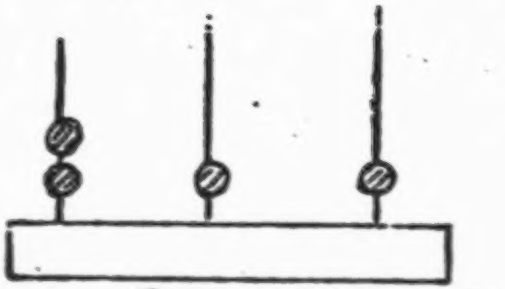
12) What is twice as much as the number shown on the abacus?

a. 211_{FIVE}

b. 222_{FIVE}

c. 224_{FIVE}

d. 422_{FIVE}



BASE FIVE ABACUS

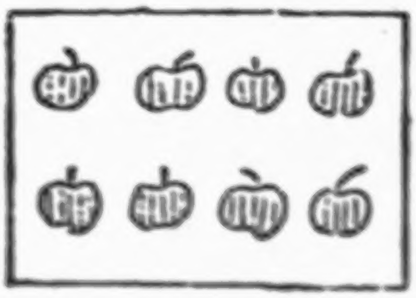
13) If 2 more apples were put in the box, how many would there be?

a. 10_{FIVE}

b. 20_{FIVE}

c. 24_{FIVE}

d. 30_{FIVE}



14) Use the table to find the difference between 12_{FIVE} and 3_{FIVE}.

a. 4_{FIVE}

b. 11_{FIVE}

c. 15_{FIVE}

d. 20_{FIVE}

+	2	3	4	10	11	12
0	2	3	4	10	11	12
1	3	4	10	11	12	13
2	4	10	11	12	13	14
3	10	11	12	13	14	20
4	11	12	13	14	20	21

BASE FIVE ADDITION

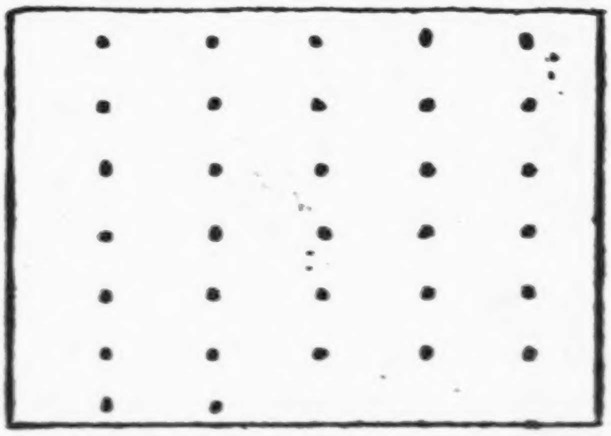
15) How many more dots are needed to make 114_{FIVE}

a. 2_{FIVE}

b. 3_{FIVE}

c. 4_{FIVE}

d. 10_{FIVE}



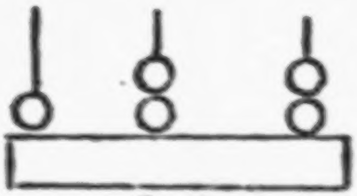
16) How much more must be added to the number on the top abacus to get the number on the lower abacus?

a. 23_{FIVE}

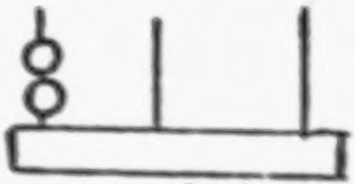
b. 33_{FIVE}

c. 100_{FIVE}

d. 122_{FIVE}



BASE FIVE ABACUS



BASE FIVE ABACUS

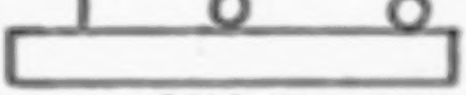
17) What base five numeral is equal to the amount shown on the base ten abacus?

a. 28_{FIVE}

b. 33_{FIVE}

c. 43_{FIVE}

d. 103_{FIVE}



BASE TEN ABACUS

18) What base ten (decimal) numeral is equal to 43_{FIVE}?

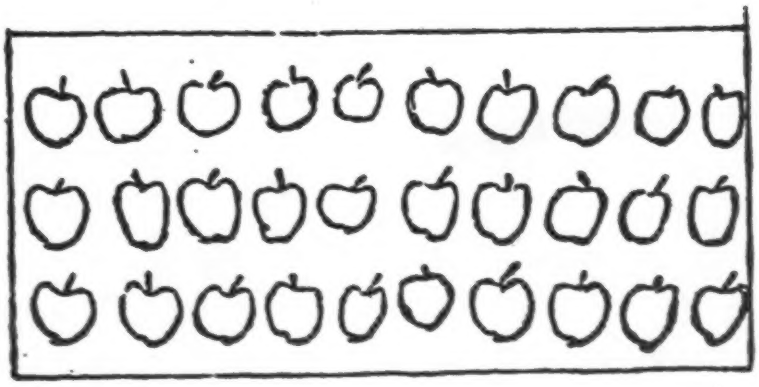
a. 19

b. 23

c. 39

d. 43

19) The box of apples is to be grouped for writing a base five numeral. Which numeral will it be?



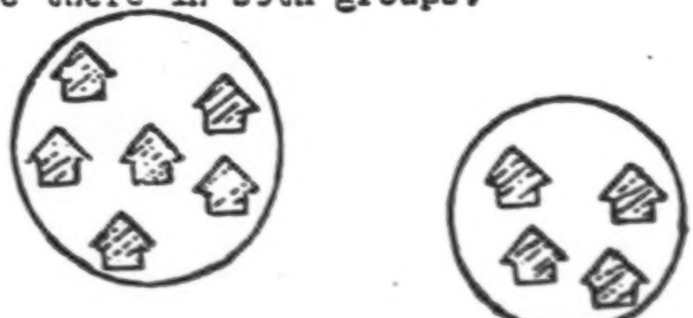
a. 15_{FIVE}

b. 30_{FIVE}

c. 60_{FIVE}

d. 110_{FIVE}

20) How many arrowheads are there in both groups?



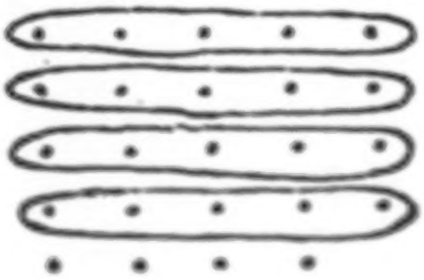
a. 14_{FIVE}

b. 20_{FIVE}

c. 64_{FIVE}

d. 114_{FIVE}

21) The picture shows a grouping of dots for the base five numeral 44_{FIVE}. What base ten (decimal) numeral would be used for the same number of dots?



a. 24

b. 42

c. 44

e. 54

22) The next base five numeral after 14_{five} is

a. 10_{FIVE}

b. 15_{five}

c. 20_{five}

d. 24_{five}

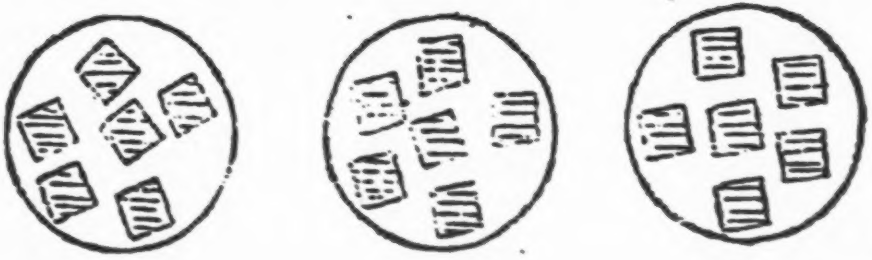
23) How many are there in the three groups?

a. 13_{FIVE}

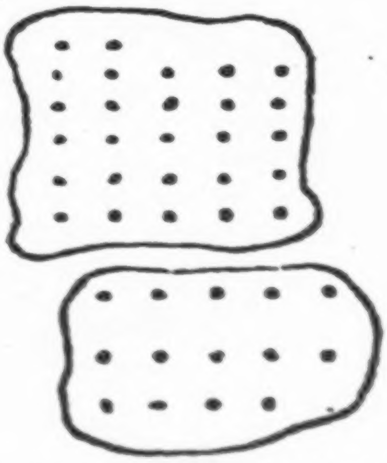
b. 31_{FIVE}

c. 33_{FIVE}

d. 40_{FIVE}



24) How many more dots are there in the top loop than in the bottom loop?



a. 4_{FIVE}

b. 20_{FIVE}

c. 22_{FIVE}

d. 23_{FIVE}

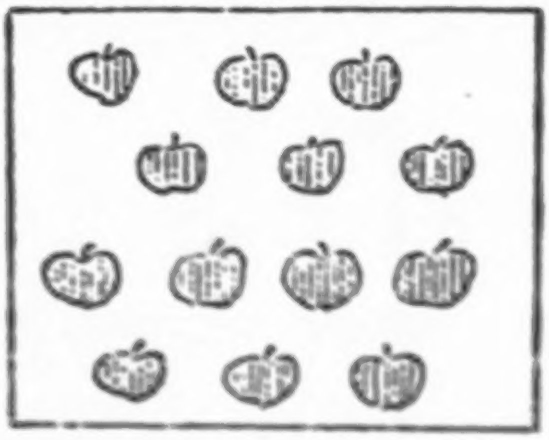
25) Which base five numeral tells how many apples are shown in the box?

a. 13_{FIVE}

b. 23_{FIVE}

c. 32_{FIVE}

d. 33_{FIVE}



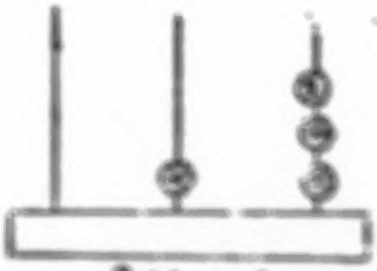
26) Putting together the amounts shown in the two pictures, how much is there in all?

a. 313_{FIVE}

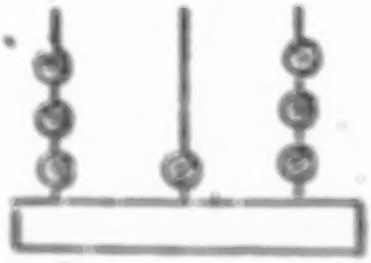
b. 326_{FIVE}

c. 331_{FIVE}

d. 336_{FIVE}



Base five abacus



Base five abacus

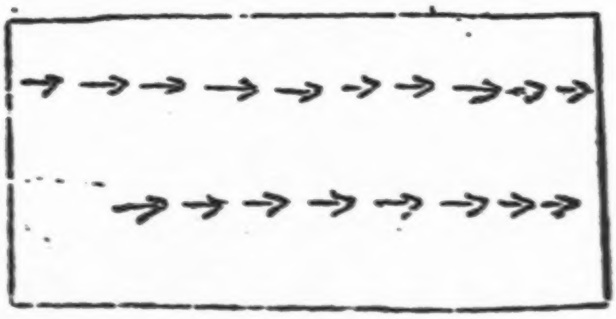
27) Group the arrows shown in the picture to be able to write the base five numeral for it. The numeral is

a. 18_{FIVE}

b. 23_{FIVE}

c. 28_{FIVE}

d. 33_{FIVE}



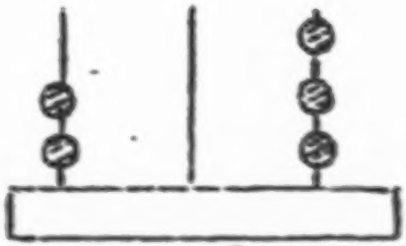
28) The abacus shows a base five numeral. Which base ten numeral has the same value as the number shown?

a. 13

b. 23

c. 53

d. 203



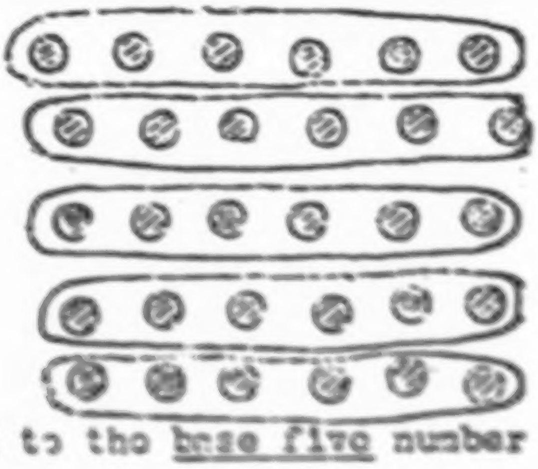
29) If 10_{FIVE} groups each contain 11_{FIVE} circles, how many are there all together?

a. 101_{FIVE}

b. 104_{FIVE}

c. 110_{FIVE}

d. 114_{FIVE}



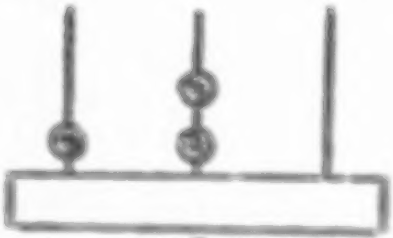
30) Which base ten number is equal to the base five number shown in the picture?

a. 7

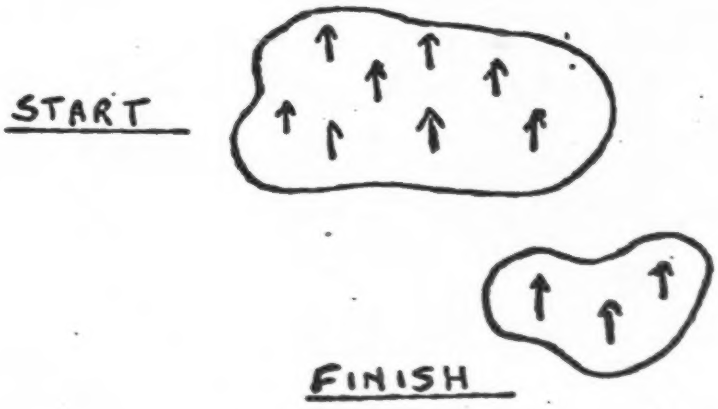
b. 12

c. 35

d. 120



31) How many arrowheads were used up, if the lower loop shows what was left?



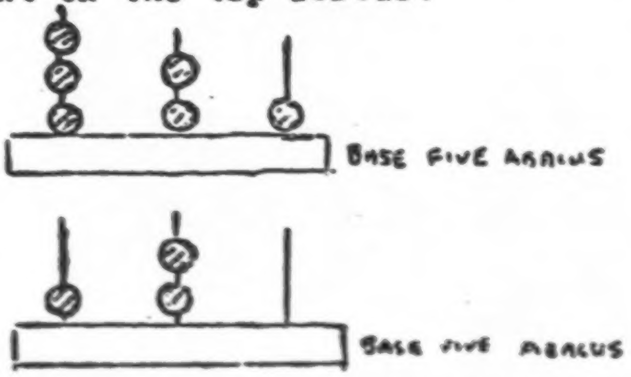
a. 3_{FIVE}

b. 5_{FIVE}

c. 10_{FIVE}

d. 11_{FIVE}

32) What is the remainder if the amount on the lower abacus is subtracted from the amount on the top abacus?



a. 21_{FIVE}

b. 102_{FIVE}

c. 104_{FIVE}

d. 201_{FIVE}

33) Mary's fishtank had 44_{FIVE} guppies in it last September. Now there are 201_{FIVE} guppies. At least how many guppies were added to the tank since last September?

a. 42_{FIVE}

b. 102_{FIVE}

c. 157_{FIVE}

d. 243_{FIVE}

34) The numeral 301_{FIVE} has the same value as which base ten (decimal) number?

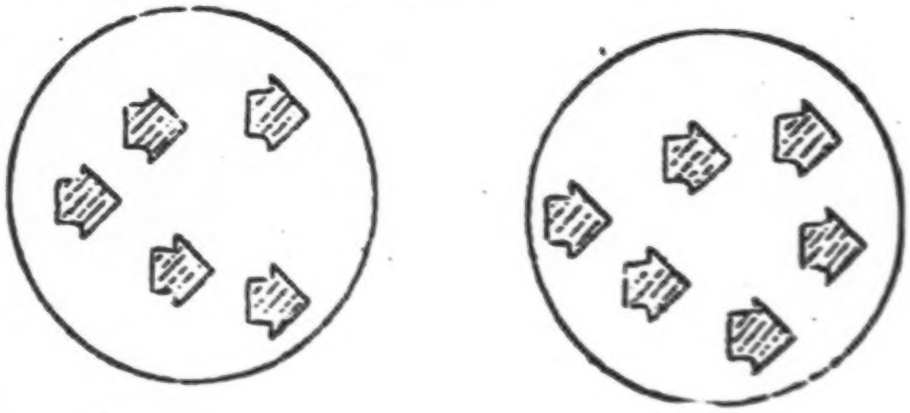
a. 16

b. 31

c. 76

d. 301

35) Choose the correct number of arrowheads in the two groups shown.



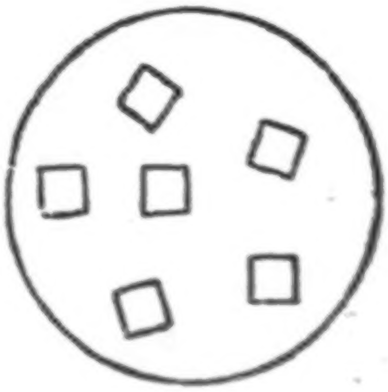
a. 11_{FIVE}

b. 12_{FIVE}

c. 21_{FIVE}

d. 22_{FIVE}

36) How many squares should be added to the group shown to make 22_{FIVE} ?



a. 4_{FIVE}

b. 10_{FIVE}

c. 11_{FIVE}

d. 13_{FIVE}

37) What is the sum of 21_{FIVE}, 31_{FIVE} , and 14_{FIVE} ?

a. 111_{FIVE}

b. 121_{FIVE}

c. 125_{FIVE}

d. 131_{FIVE}

38) What number added to 114_{FIVE} makes 244_{FIVE} ?

a. 30_{FIVE}

b. 40_{FIVE}

c. 130_{FIVE}

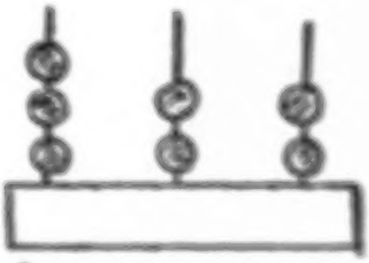
d. 131_{FIVE}

39) John has 22_{five} baseball tickets. If Billy gives him 21_{five} more, how many does he have then?

- a. 22_{five}
- b. 22
- c. 43_{five}
- d. 43

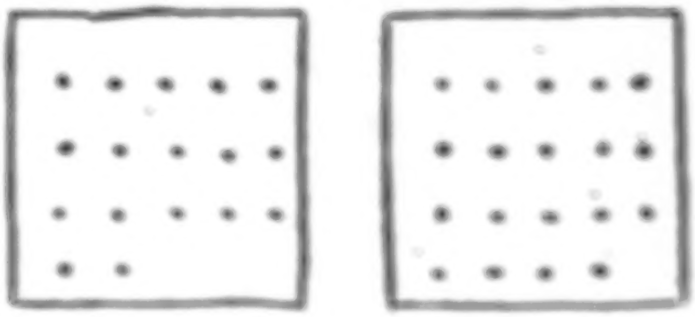
40) Starting with the base five abacus shown, how many beads must be added to show the number 342_{five}?

- a. 1_{five}
- b. 2_{five}
- c. 10_{five}
- d. 20_{five}


BASE FIVE ABACUS

41) What is the sum of the two groups of dots shown?

- a. 34_{five}
- b. 71_{five}
- c. 121_{five}
- d. 211_{five}



42) The number 31_{five} is equal to which base ten numeral?

- a. 16
- b. 31
- c. 61
- d. 101

APPENDIX D

NON-DECIMAL TEST - GRADE SIX

NON-DECIMAL TEST GRADE SIX

Read directions carefully.

This is a multiple choice test.

For each question, only ONE answer is correct.

Read each question carefully. Do your figuring on the question sheet. Then select the right answer from the choices given.

Find the number of the question on the answer card.

Circle the letter which goes with the answer you have chosen.

SAMPLE

1. How much is $2_{\text{five}} + 2_{\text{five}}$?

a. 3_{five}
b. 4_{five}
c. 11_{five}
d. 12_{five}

The correct answer is b. Look on your answer card to see the circle around b.

1. a [✓ b] c d

When told to do so, begin to work each question, marking your answer sheet as you go along. Take your time, work carefully, and try not to make wild guesses.

Some easy questions come after some hard questions.

Do not leave your seat. If you have any questions, raise your hand and the teacher will come to your seat.

When you finish each page, go on to the next.
DO NOT TURN BACK.

When you reach the end of the test, turn the test booklet over on your desk. Cover the answer sheet and sit quietly.

THE MARGINAL LEGIBILITY OF THIS PAGE IS DUE TO POOR ORIGINAL COPY. BETTER COPY WAS NOT AVAILABLE AT THE TIME OF FILMING. E.D.R.S.

2) Which numbers should be used to complete the table shown below?

a. 7 and 15_{FIVE}

b. 7_{FIVE} and 15_{FIVE}

c. 7_{FIVE} and 20_{FIVE}

d. 7 and 20_{FIVE}

4	4_{FIVE}
5	10_{FIVE}
6	11_{FIVE}
▨	12_{FIVE}
8	13_{FIVE}
9	14_{FIVE}
10	▨

3) Double the number shown on the abacus.

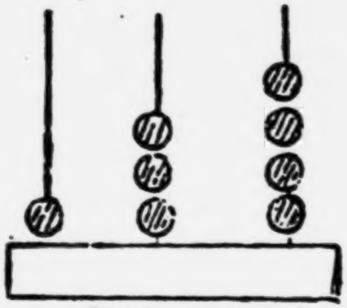
Your answer is

a. 214_{FIVE}

b. 268_{FIVE}

c. 323_{FIVE}

d. 343_{FIVE}



BASE FIVE ABACUS

4) $103_{\text{FIVE}} \times 22_{\text{FIVE}} =$ ________

a. 422_{FIVE}

b. 2266_{FIVE}

c. 2311_{FIVE}

d. 2321_{FIVE}

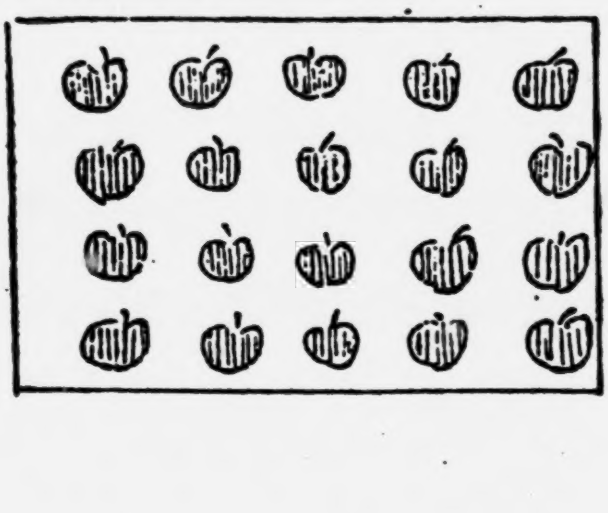
5) If 10_{FIVE} more apples were put into the box, how many would be inside?

a. 21_{FIVE}

b. 31_{FIVE}

c. 51_{FIVE}

d. 100_{FIVE}



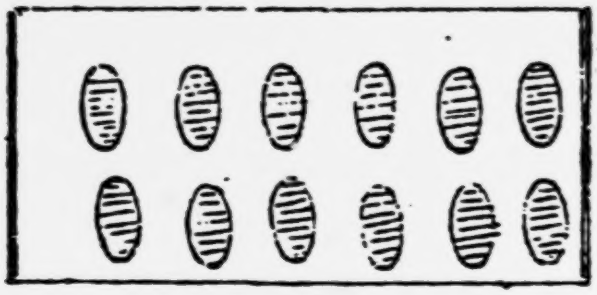
6) If 3 eggs were removed from the group of eggs in the box, how many would be left?

a. 9_{FIVE}

b. 14_{FIVE}

c. 21_{FIVE}

d. 22_{FIVE}



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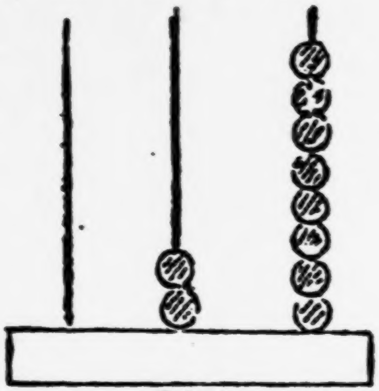
7) What base five numeral is equal to the amount shown on the base ten abacus?

a. 28_{FIVE}

b. 43_{FIVE}

c. 53_{FIVE}

d. 103_{FIVE}



BASE TEN ABACUS

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8) Use the table to find the difference between 12_{FIVE} and 3_{FIVE}

a. 4_{FIVE}

b. 13_{FIVE}

c. 14_{FIVE}

d. 20_{FIVE}

+	2	3	4	10	11	12
0	2	3	4	10	11	12
1	3	4	10	11	12	13
2	4	10	11	12	13	14
3	10	11	12	13	14	20
4	11	12	13	14	20	21

BASE FIVE ADDITION

---

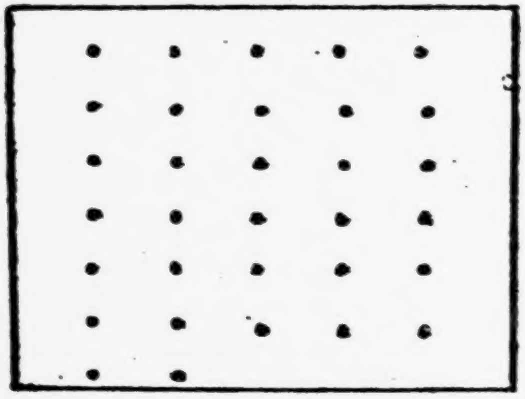
9) How many more dots are needed to make 114_{FIVE}?

a. 2_{FIVE}

b. 10_{FIVE}

c. 12_{FIVE}

d. 22_{FIVE}



10) Which base ten (decimal) numeral is equal to 100_{FIVE}?

a. 20

b. 25

c. 100

d. 500

---

11) Fill in the missing number in the table shown below.

a. 2246_{FIVE}

b. 2321_{FIVE}

c. 2351_{FIVE}

d. 2401_{FIVE}

×	100	101	102
20	2000	2020	2040
21	2100	2121	2142
22	2200	2222	2244
23	2300	2323	▨

BASE FIVE MULTIPLICATION

---

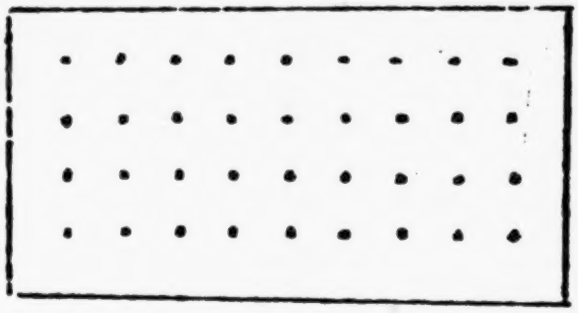
12) How many dots are there in 4_{FIVE} groups each containing 14_{FIVE} dots?

a. 44_{FIVE}

b. 64_{FIVE}

c. 114_{FIVE}

d. 121_{FIVE}



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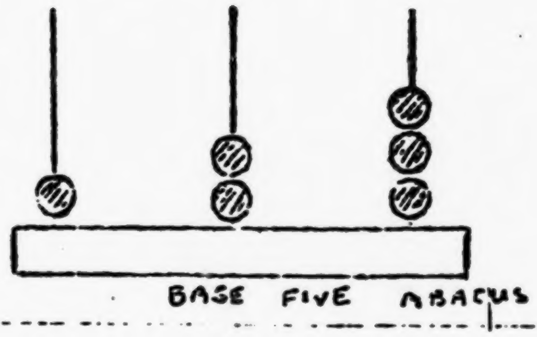
13) What is 12_{FIVE} times the number shown on the abacus?

a. 130_{FIVE}

b. 1476_{FIVE}

c. 2012_{FIVE}

d. 2031_{FIVE}



14) Which base ten (decimal) numeral has the same value as 110_{FIVE} ?

a. 26

b. 30

c. 55

d. 135

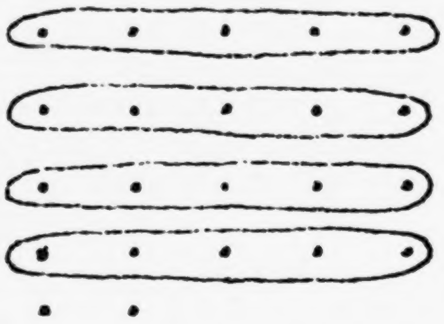
15) The picture shows a grouping of dots for the base five numeral 44_{FIVE} . What base ten (decimal) numeral would be used for the same number of dots?

a. 14

b. 24

c. 44

d. 54



16) $104_{\text{FIVE}} + 142_{\text{FIVE}} =$ ____________

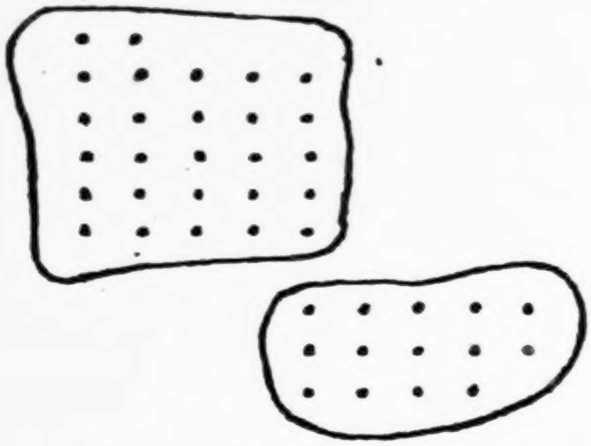
a. 242_{FIVE}

b. 246_{FIVE}

c. 301_{FIVE}

d. 311_{FIVE}

17) How many more dots are there in the top loop than in the bottom loop?



a. 4_{FIVE}

b. 20_{FIVE}

c. 23_{FIVE}

d. 33_{FIVE}

18) The number 31_{FIVE} is equal to which base ten numeral?

a. 16

b. 21

c. 31

d. 61

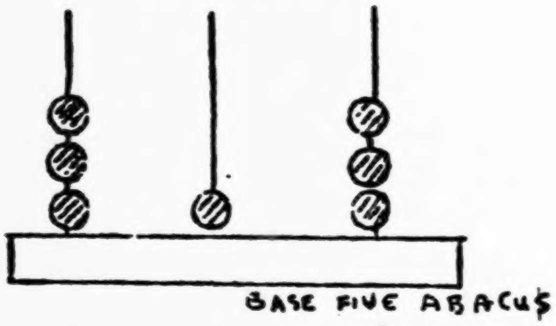
19) Putting together the amounts shown in the two pictures, how much is there in all?

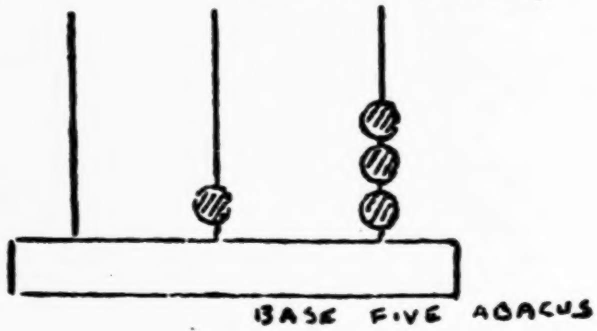
a. 313_{FIVE}

b. 326_{FIVE}

c. 331_{FIVE}

d. 336_{FIVE}





20) How does the number of digits in a base five numeral compare with the number in the equal base ten numeral?

a. Sometimes, there are more in the base five numeral.

b. Always, there are more in the base five numeral.

c. Sometimes, there are more in the base ten numeral.

d. Always, there are more in the base ten numeral.

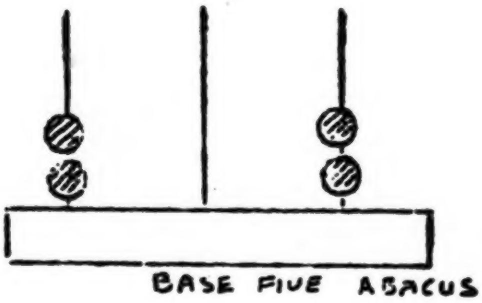
21) The abacus shows a base five numeral. Which Base ten numeral has the same value as the number shown?

a. 22

b. 52

c. 202

d. 252



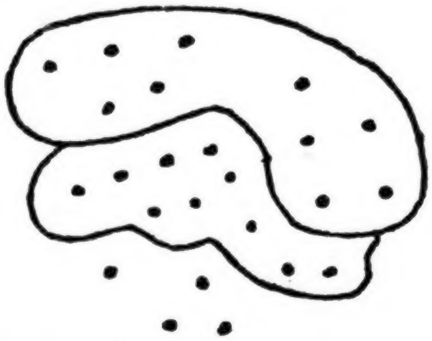
22) Which is the base ten (decimal) numeral for the number of dots in the diagram?

a. 14

b. 24

c. 44

d. 104



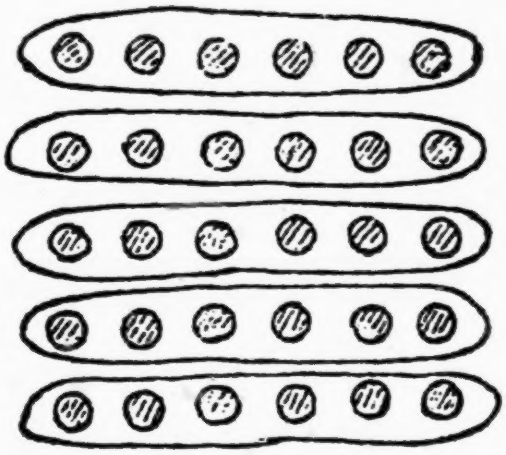
23) If 10_{FIVE} groups each contain 11_{FIVE} circles, how many are there all together?

a. 65_{FIVE}

b. 101_{FIVE}

c. 110_{FIVE}

d. 114_{FIVE}



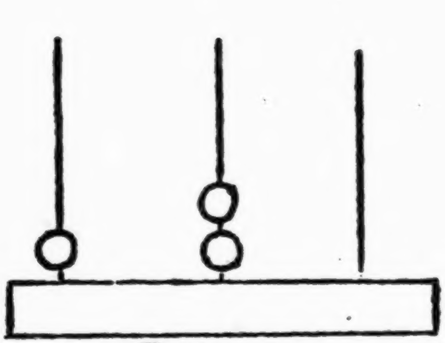
24) Which base ten number is equal to the base five number shown in the picture?

a. 7

b. 12

c. 35

d. 120



BASE FIVE ABACUS

25) Use the table shown to figure out $22 \div 3 =$ ______.

a. 2_{FIVE}

b. 4_{FIVE}

c. 41_{FIVE}

d. 121_{FIVE}

x	4	10	11	12	13	14	20	21	22
2	13	20	22	24	31	33	40	42	44
3	22	30	33	41	44	102	110	113	121
4	31	40	44	103	112	121	130	134	143

BASE FIVE MULTIPLICATION

26) What base ten (decimal) numeral is equal to 43_{FIVE}?

a. 18

b. 23

c. 28

d. 43

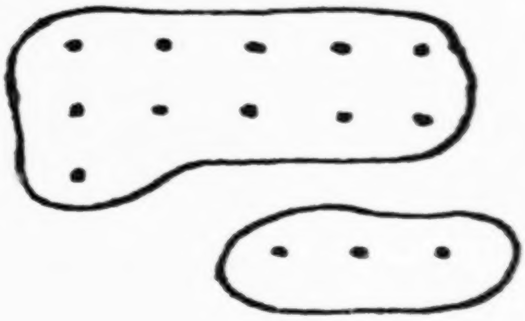
27) How many dots are in the two loops shown below:

a. 13_{FIVE}

b. 14_{FIVE}

c. 23_{FIVE}

d. 24_{FIVE}



28) Which number is 10 more than 190?

a. 180

b. 200

c. 290

d. 1910

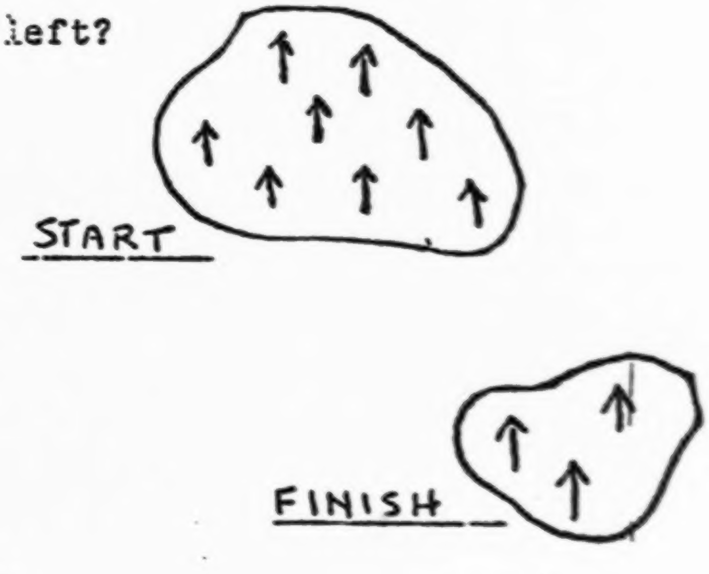
29) How many arrowheads were used up, if the lower loop shows what was left?

a. 3_{FIVE}

b. 5_{FIVE}

c. 10_{FIVE}

d. 11_{FIVE}



30) How many dots are in a picture showing 3_{FIVE} rows each containing 11_{FIVE} dots?

a. 13_{FIVE}

b. 31_{FIVE}

c. 33_{FIVE}

d. 103_{FIVE}

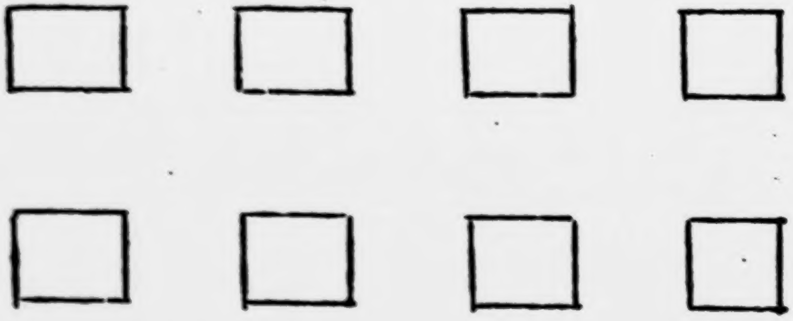
31) Each box shown below contains 13_{FIVE} pieces of candy. How many pieces of candy are in all the boxes shown below?

a. 13_{FIVE}

b. 104_{FIVE}

c. 169_{FIVE}

d. 224_{FIVE}



32) Which of the following is the correct choice of the missing number in $3682 = 3000 + 600 + \_\_\_\_ + 2$

a. 8

b. 80

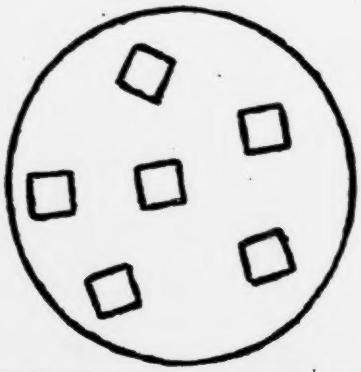
c. 88

d. 800

33) The numeral 301_{FIVE} has the same value as which base ten (decimal) numeral?

a. 16
b. 31
c. 76
d. 301

34) How many squares should be added to the group shown to make 22_{FIVE} ?

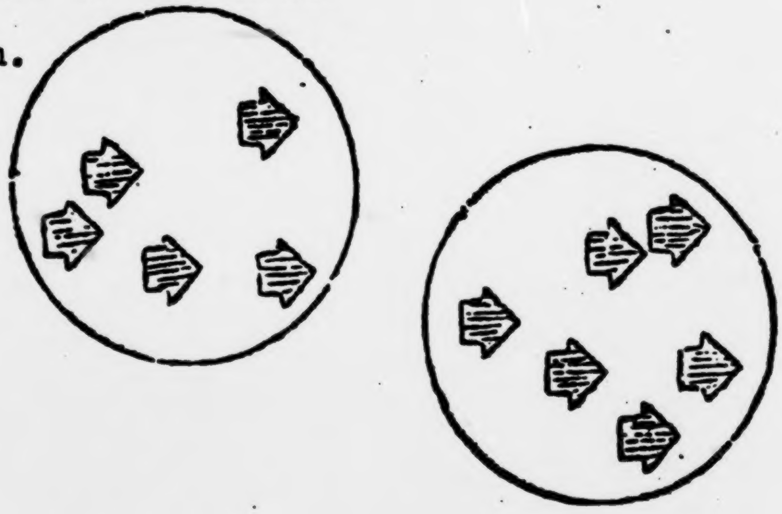


a. 10_{FIVE}
b. 11_{FIVE}
c. 12_{FIVE}
d. 13_{FIVE}

35) What is the sum of 21_{FIVE}, 31_{FIVE}, and 14_{FIVE} ?

a. 121_{FIVE}
b. 122_{FIVE}
c. 131_{FIVE}
d. 141_{FIVE}

36) Choose the correct number of arrowheads in the two groups shown.



a. 11_{FIVE}
b. 13_{FIVE}
c. 21_{FIVE}
d. 23_{FIVE}

37) Fill in the missing numbers:

$$2013_{FIVE} = 2000_{FIVE} + \_\_\__{FIVE} + \_\_\__{FIVE}$$

a. 10 and 3

b. 100 and 30

c. 100 and 3

d. 1000 and 3

38) Express the number 111 as a base five numeral:

a. 11_{FIVE}

b. 31_{FIVE}

c. 421_{FIVE}

d. 555_{FIVE}

39) $312_{FIVE} + 211_{FIVE} - 121_{FIVE} = \_\_\_\_$

a. 382_{FIVE}

b. 402_{FIVE}

c. 644_{FIVE}

d. 1144_{FIVE}

40) 10 is equal to which base five numeral?

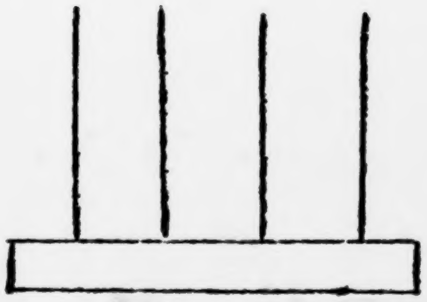
- a. 5
- b. 5_{FIVE}
- c. 10_{FIVE}
- d. 20_{FIVE}

41) What is the product of 3001_{FIVE} and 20_{FIVE}

- a. 2431_{FIVE}
- b. 3021_{FIVE}
- c. $11{,}002_{\text{FIVE}}$
- d. $110{,}020_{\text{FIVE}}$

42) How many beads would be needed to show the sum of 312_{FIVE} and 132_{FIVE} on a base five abacus?

- a. 4_{FIVE}
- b. 12_{FIVE}
- c. 22_{FIVE}
- d. 444_{FIVE}



BASE FIVE ABACUS

APPENDIX E

DESCRIPTION OF IDENTIFICATION DATA USED IN STUDY

Description of 37 Variables Recorded on IBM Data Cards

Variable	Description	Column Number
1	Class Affiliation	4,5
2	Sex	6
3	Method	7
4	Grade Level	8
5	School	9
6	Teacher Experience	10
7	Race	11
8	Socio-Economic Background	12
9	Age in months	13,14,15
10	Former IQ scores	16,17,18
11	Reading Level-Teacher Estimate	24
12	Arithmetic Level-Teacher Estimate	25
13	Visual Pre-Test Score	26,27
14	Auditory Pre-Test Score	28,29
15	Motor Coordination Score	30,31
16	CTMM Pretest	32,33,34
17	CTMM Posttest II	35,36,37
18	STAN-Test 1, Form X Pretest Computation	38,39
19	STAN-Test 2, Form X Pretest Arithmetic Reasoning	40,41
20	STAN-Test 1, Form W Posttest I	42,43
21	STAN-Test 2, Form W Posttest I	44,45
22	STAN-Test 1, Form X Posttest II	46,47
23	STAN-Test 2, Form X Posttest II	48,49
24	Non-Decimal Test Posttest I	50,51
25	Non-Decimal Test Posttest II	52,53
26	Geometry Test Posttest I	54,55
27	Place Value Subtest-STAN Form X Pretest Score to 8	
28	Place Value Subtest-STAN Form W Psttest I Score to 8	
29	Place Value Subtest-STAN Form X Psttest II Score to 8	
30	V20-V18	
31	V21-V19	
32	V22-V18	
33	V23-V19	
34	V25-V24	
35	V28-V27	
36	V29-V28	
37	V29-V27	

APPENDIX F

TEST OF SIGNIFICANCE OF THE CORRELATION COEFFICIENT

Significance of the Correlation

The relative ease and speed of electronic computer have made possible the calculation of large numbers of intercorrelations for variables in statistical experiments. Not all correlation coefficients are sufficiently different from zero to doubt the independence of a pair of variables under consideration.

Critical points for the acceptance regions of the correlation coefficient based on the assumption of bivariate normality were computed by using the values of the t-distribution (Dixon-Massey, 1957).[1]

Table 124 shows a table adapted for use with the group frequencies in this experiment.

TABLE 124

99% Critical Values for the Correlation Coefficient r,[2] when $\rho = 0$ and n=number of pairs

n	r	n	r	n	r	n	r
19	.575	29	.470	53	.350	105	.248
20	.561	30	.463	54	.346	107	.245
21	.549	31	.455	61	.327	128	.222
22	.531	32	.449	66	.316	136	.216
23	.526	33	.440	70	.307	201	.180
24	.515	34	.433	71	.305	208	.175
25	.505	35	.422	93	.266	222	.171
26	.496	41	.397	94	.263	229	.170
27	.489	42	.393	96	.261	327	.140
		45	.376	101	.254	357	.134
		48	.360				
28	.479	52	.354				

---

[1] On a two-tailed test where level of significance $\alpha = .01$

[2] Adapted from Table A-30a, Dixon and Massey, 1957, p.468., and Table 13 Pearson and Hartley, 1962, p.138; with missing values computed from percentiles of t by the relation $r = t/\sqrt{t^2 + N-2}$

APPENDIX G

KOLMOGOROV - SMIRNOV TWO-SAMPLE TEST

Kolmogorov-Smirnov Two-Sample Test

The Kolmogorov-Smirnov Two-Sample Test is a test of whether two independent samples have been drawn from the same sample population or from populations having similar distributions (Siegel, 1956, p. 127).

For the case of large samples as in this instance n_1 and n_2 may be unequal.

For a one-tailed test, $D = \max S_{n_1}(X) - S_{n_2}(X)$ is used in the following formula based on the Chi Square relation:

$$X^2 = 4D^2 \frac{n_1 n_2}{n_1 + n_2}$$

This has a sampling distribution approximated by the Chi Square distribution with two degrees of freedom.

The symbols used above are:

$S_{n_1}(X)$ = the observed cumulative step function of the first sample

$S_{n_2}(X)$ = the observed cumulative step function of the second sample

D = the signed maximum difference of any step.

APPENDIX H

HARTLEY MAX-F TEST OF HOMOGENEITY OF VARIANCE

TABLE 125

Percentage Points of Ratio (s^2max/s^2min) for k=4
Hartley-Maximum F Test

Degrees of Freedom	Upper 5% Point	Upper 1% Point
30	2.61	3.4
42	2.24	2.8
43	2.21	2.7
45	2.18	2.7
47	2.14	2.6
48	2.12	2.6
52	2.06	2.5
53	2.05	2.4
54	2.03	2.4
60	1.96	2.3
66	1.87	2.2
70	1.82	2.1
Inf.	1.00	1.0

TABLE 126

Hartley Max-F Test for Difference Scores and Place Value Subtest Scores of Arithmetic Achievement Tests

Title	Var. Max	Var. Min	n_1n_2	$\frac{s^2max}{s^2min}$
Place Value Subtest				
Grade Four				
Pretest	1.5785	1.2631	43,48	1.562
Posttest I	1.9651	1.6953	43,42	1.343
Posttest II	1.4464	1.3525	52,48	1.144
Difference Scores				
STAN Test I				
Posttest I-Pretest	5.6121	3.8216	43,52	1.720
Psttest II-Psttest I	5.3950	4.1807	43,48	1.665
STAN Test II				
Posttest I-Pretest	4.5400	3.3608	43,52	1.817
Psttest II-Psttest I	3.9750			
Place Value Subtest				
Grade Six				
Pretest	1.7742	1.5738	54,45	1.271
Posttest I	2.2679	1.8601	45,53	1.487
Posttest II	2.1245	1.6717	47,53	1.615
Difference Scores				
STAN Test I				
Posttest I-Pretest	4.876	3.923	47,54	1.545
Psttest II-Psttest I	4.490	3.1048	47,53	2.092**
STAN Test II				
Psttest I-Pretest	5.5153	3.8881	47,54	2.012
Psttest II-Psttest I	5.0158	2.7463	47,53	3.336***

APPENDIX I

SCHEFFE TEST OF COMPARISON OF MEANS

Scheffe Test for Comparisons Between Means

Possible Comparisons Between Treatment Means

	$\bar{X}_1$	$\bar{X}_2$	$\bar{X}_3$	$\bar{X}_4$	a_i^2	d_i	s_{di}[1]	t[2]
(1)vs(2)	1	-1	0	0	2	$\bar{X}_1-\bar{X}_2$		
(1)vs(3)	1	0	-1	0	2	$\bar{X}_1-\bar{X}_3$		
(1)vs(4)	1	0	0	-1	2	$\bar{X}_1-\bar{X}_4$		
(2)vs(3)	0	1	-1	0	2	$\bar{X}_1-\bar{X}_3$		
(2)vs(4)	0	1	0	-1	2	$\bar{X}_2-\bar{X}_4$		
(3)vs(4)	0	0	1	-1	2	$\bar{X}_3-\bar{X}_4$		
(2)vs(1)+(3)+(4)	-1	3	-1	-1	12	$\bar{X}_2-\frac{(\bar{X}_1+\bar{X}_3+\bar{X}_4)}{3}$		
(1)vs(3)+(4)	2	0	-1	-1	6	$X_1-\frac{(X_3+X_4)}{2}$		

---

[1] $s_{di} = \sqrt{S^2\left(\frac{a_{1i}^2}{n_1}+\frac{a_{2i}^2}{n_2}+\ldots+\frac{a_{ki}^2}{n_k}\right)}$ where S^2 is the error mean square of the analysis of variance. This may be replaced by

$s_{di} = \sqrt{\frac{S^2}{n}\sum a_i^2}$, when group numbers are equal (Edwards, 1962, p. 142).

[2] $t = \frac{d_i}{s_{di}}$

Scheffe Test - t' Values[1]

$t' = \sqrt{(k-1)\ F}$ by definition

$k = 4$, in this experiment

F = tabled values which in the case of unequal groups has $(k-1)$ df in the numerator and

$$\sum_{i=1}^{k} (n_i-1) \text{ df}$$

in the denominator.

$\therefore$ In this experiment for the fourth grade means, we used F (3,182) and for the sixth grade means, we used F (3,195). These are most closely approximated by tabled values for F (3, 200).

Values of F (3,200)[2]

Percentage Point	F (3,200)	$t' = \sqrt{(k-1)F}$	$\lvert t \rvert \geqslant t'$
.75	1.38	2.27	#
.90	2.11	2.52	*
.95	2.65	2.82	**
.99	3.88	3.41	***

---

[1] Allen L. Edwards. Statistical Methods. New York: Holt, Rinehart Winston, 1967, p. 266.

[2] B.J. Winer. Statistical Principles in Experimental Design. New York: McGraw Hill. 1962, p. 646.

According to Winer (1962) in his comparison of the Scheffe, Tukey, Newman-Keuls and Duncan methods, he stated:

> The Scheffe method is clearly the most conservative with respect to Type 1 error; This method will lead to the smallest number of significant differences. In making tests on differences between all possible pairs of means it will yield too few significant results. (p. 89)

It should be pointed out that the test is so constructed that the probability that all statements concerning the significance are true is equal to or greater than 1 - α . Thus if $\alpha = .05$, the probability that all statements made will be correct is $\geqslant .95$. (Edwards, 1962, p. 155)

As a consequence, larger differences will be required for significance. Scheffe suggested with his test one might consider taking $\alpha = .10$ rather than $\alpha = .05$ (Edwards, p. 154).

As a further guarantee against stating that means are significantly different when in fact they are not, the $\underline{n}$ of these unequal groups was here assumed to be the smallest $\underline{n}_i$, that which produces the fewest significant t's.

In view of the conservative nature of the test, the .25 level of confidence as well as the .10, .05, .01 were indicated by the symbols shown. For the .25 level of confidence, the probability of the hypothesis being correctly accepted is at least 75%.

APPENDIX J

SAMPLE QUESTIONS ON PLACE VALUE SUBTEST

GRADE FOUR

17 Which is eight thousand sixteen?

a 80,016 c 8016
b 8,16 d 800,016

a b c d
17 ○ ○ ○ ○

16 Which is eight thousand ninety-two?

e 892 g 8092
f 800,092 h 8902

e f g h
16 ○ ○ ○ ○

GRADE SIX

7 In which of the following has the 4 the greatest value?

a 48.36 c 34.57
b 432 d 82.47

a b c d
7 ○ ○ ○ ○

12 In which of the following has the 6 the greatest value?

e 64 g 6.432
f 3.46 h 56

e f g h
12 ○ ○ ○ ○